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MANEUVER DETECTION USING DESCRIPTIVE REGULARIZATION

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RESUME

On traite ici le problème de la détection de manoeuvre qui se produit dans l'analyse de la motion de véhicule comme un problème inverse improprement posé de l'estimation du signal. On définit la manoeuvre comme l'occurence d'une variation considerable et localisée de la courbure du trajectoire. Du fait que ce problème soit improprement posè, l'evaluation directe numérique de la courbure en fonction du temps ou de la distance impliquerait la différentiation repetée des données du trajectoire, possiblement en présence du bruit à haut niveau.

Afin de contourner ces difficultés, une technique convenable de régularisation, utilisant l'information à priori tant sur les statistiques que sur la forme du signal à être recupere, fut développée. On definit un problème variationel en utilisant une formule de la vraisemblance maximum sous les statistiques due signal et du bruit. L'information à priori de la forme du signal est utilisée pour imposer des constraintes sur la solution. Un algorithme iteratif est presenté afin de résoudre le problème de l'optimisation convex contrainte qui en résulte. L'algorithme pourvoit aux besoins des deux assomptions fondamentales sur la forme du signal à être recupéré.

Les applications pourraient comprendre les moyens d'éviter les collisions de navires, le repérage passif et le contrôle du traffic aérien.

SUMMARY

The problem of maneuver detection which arises in vehicle motion analysis is treated here as an ill-posed inverse problem of signal estimation. The maneuver is defined as an occurrence of a considerable and localized variation of the vehicle path curvature. Due to the ill-posedness of this problem, direct numerical evaluation of the curvature as a function of time or range would imply the repeated numerical differentiation of the path data, possibly in the presence of the high-level noise.

To obviate these difficulties, an appropriate regularization technique was developed which utilizes the a priori information both on the statistics and on the shape of the signal to be recovered. A variational problem is defined by a maximum likelihood statement under specific statistics of the signal and the noise. The shape information is utilized in the form of constraints on the solution. A finite iterations algorithm is used to solve the resulting constrained convex optimization problem. The algorithm provides for two basic assumptions on the shape of the signal to be recovered.

The problem of erroneous or incomplete a priori information is addressed. Applications may include ship collision avoidance, passive tracking, and air-traffic control.

I. Introduction

In vehicle motion analysis (VMA), a maneuver can be defined as an occurrence of a considerable and localized variation of the vehicle path curvature. Maneuver detection, especially on an on-line basis, is important in the problems of ship collision avoidance, air traffic control, passive tracking, etc., and therefore has attracted the attention of researchers in the framework of VMA. A standard approach to the problem is exemplified by [1] where a statistical analysis based on the Kalman filter approach is utilized.

A suitable hypothesis on the maneuver or a number of hypotheses are normally assumed in VMA. Typical maneuvers on a plane are shown on Fig. 1.

The variation of the path curvature at the moment of the maneuver is, for case 1,a ("zig maneuver"), a narrow pulse ("spike"). In the cases 1,b and 1,c the curvature undergoes a step variation.

Since the path curvature is adequately described by the second derivative of the path, whether in rectangular or polar coordinates, or in parametric form, an obvious alternative approach to [1] and similar works is the direct recovery of the second derivative of the path.

According to this approach, the problem is represented as an inverse problem in linear spaces X and Y of real-valued functions:

$$\begin{aligned} &Kz(t) = u(s) \\ &K: X \to Y \end{aligned} \tag{1.1}$$

with u being the vector of path data, z its second derivative and K the map of double integration. As a Hilbert-Schmidt integral operator, this map is

$$Kz(t) = \int_0^t (t-s)u(s)ds. \tag{1.2}$$

This approach, however, presents obvious difficulties. Since the operator K has an unbounded inverse, the problem represented by equation (1.1) is an ill-posed problem of the type found in signal processing theory and related fields [2-4]. It relates to the well-known fact that if data are contaminated by heavy noise, as is usually the case in practice, the recovery of the second derivative presents formidable difficulties. Probably, for these reasons, this avenue for maneuver detection has remained unexplored.

Recently, though, some efficient approaches known under the collective name of regularization methods have been developed to deal with ill-posed problems. In view of the availability of such techniques, the solution for the inverse problem of maneuver detection no longer seems unrealistic.

Any regularization method can be interpreted as the utilization of some information available (or assumed) a priori about the solution. The usual approach to regularization consists of imposing smoothness conditions, either deterministic or statistical. This way alone is, however, not as efficient as a new regularization approach, namely, descriptive regularization, which requires a priori information on the shape of the solution to the inverse problem (1.1). This requirement fits well into the specifics of the maneuver detection problem.

In [5], a comprehensive method of descriptive regularization was suggested which utilizes constraints on both the statistical smoothness of the solution and the shape (conditions of non-negativity, or monotonicity, or convexity, etc.). Such shape constraints assign the solution to a compact set in X, thus turning the problem into a well posed one, and the smoothness conditions in their turn contribute to the obtaining of a reliable solution by taking into account the known statistical properties of the signal and the noise. In this paper, we present the application of the method developed in [5] to the case of maneuver detection.

2. Descriptive Regularization for Zig Detection

We will consider the finite-dimensional approximation of equation (1.1)

$$K\underline{z} = \underline{\tilde{u}}, \tag{2.1}$$

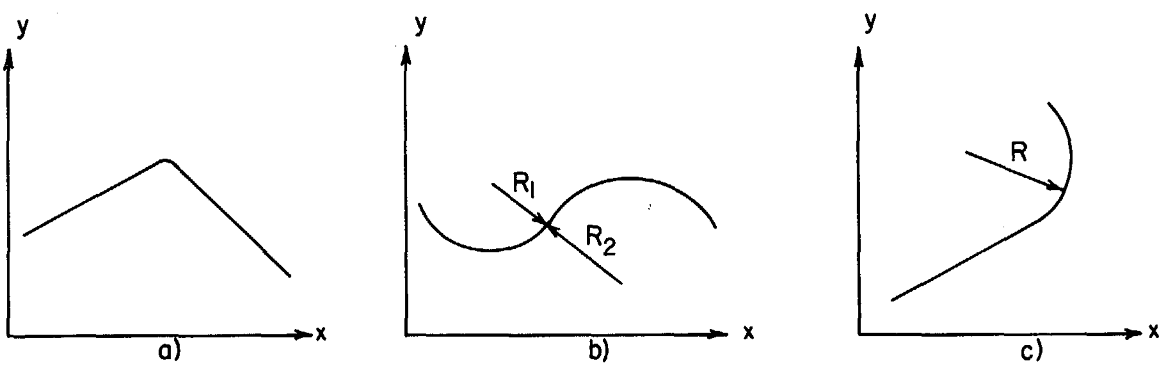

Fig. 1

where the values of the observed path data vector $\tilde{u}\varepsilon R^m$ are corrputed by the additive noise vector $w\varepsilon R^m$

$$\tilde{u} = u + w \, , \qquad (2.2)$$

the solution is represented as the vector $z\varepsilon R^n$, and K is the m x n matrix

$$K = \frac{1}{m}\begin{pmatrix} 1 & 0 & 0 & 0 & . & . & 0 \\ 2 & 1 & 0 & 0 & . & . & 0 \\ 3 & 2 & 1 & 0 & . & . & 0 \\ 4 & 3 & 2 & 1 & . & . & . \\ . & . & . & . & . & . & . \\ . & . & . & . & . & . & . \\ n & n-1 & n-2 & n-3 & . & . & 1 \\ . & . & . & . & . & . & . \\ m & m-1 & m-2 & m-3 & . & . & . \end{pmatrix} \qquad (2.3)$$

The corresponding normal equation is

$$K^TKz = K^T\tilde{u} \; . \qquad (2.4)$$

We will assume the noise normal and unbiased, $w\varepsilon N(0,\sigma)$, with u and w uncorrelated, and the value of standard deviation σ of the noise specified. The variance matrix of the solution z,

$$C = [c_{ij}] , \qquad (2.5)$$

will also be assumed given.

If, at this stage, we restrict ourselves to the case of zig maneuver, we know a priori that the solution has the shape of a narrow pulse ("spike"). A less restrictive condition turns out to be sufficient for the efficient regularization of the problem:

$$z_i \geq 0, \qquad i = 1,2,\ldots,n \, , \qquad (2.6)$$

or

$$z_i \leq 0, \qquad i = 1,2,\ldots,n \, . \qquad (2.7)$$

The method of maneuver detection consists in checking both the hypotheses (2.6) and (2.7), i.e. the "solutions" to (2.1) are found under such constraints. If a "solution" is identically zero, then the corresponding hypothesis is untrue. If both versions yield a zero "solution", then no maneuver occured in the interval considered. Depending on the relative importance of obtaining fast results versus reducing the complexity of the software, the hypotheses can be tried sequentially or in parallel.

Having this in mind, we will consider the following variational problem:

find $\hat{z}$, $\hat{z}\varepsilon R^n$, such that

$$M(\hat{z}) = \inf M(z) \qquad (2.8)$$

and

$$M(z) = (z,Dz) - 2(K^T\tilde{u},z) \, , \qquad (2.9)$$

$$D = K^TK = \sigma^2C^{-1} \, . \qquad (2.10)$$

It can be shown that this variational problem yields the maximum likelihood estimate to the problem (2.1)

$$\hat{z} = (K^TK + {}^2C^{-1})^{-1}K^T\tilde{u} \, , \qquad (2.11)$$

i.e. the regularized solution to (2.1) under the conditions of statistical smoothness.

To use the shape information, we will modify this problem by restricting $\hat{z}$ to one of the classes

$$E^{+} := \{\underline{z} : \underline{z} \epsilon R^{n}, \underline{z} \geq 0\} \quad (2.12)$$

or

$$E^{-} := \{\underline{z} : \underline{z} \epsilon R^{n}, z \leq 0\} , \quad (2.13)$$

and thus reformulating the variational problem:

find $\hat{\underline{z}}$, $\hat{\underline{z}} \epsilon R^{n}$, such that

$$M(\hat{\underline{z}}) = \inf M(\underline{z}) \quad (2.14)$$

$$\hat{\underline{z}} \epsilon E , \quad (2.15)$$

where $E = E^{+}$ or $E = E^{-}$.

Minimization of $M(\underline{z})$ under constraints (2.15) is a quadratic optimization problem. To solve this problem, a finite iteration procedure was developed [8] which consists of the selection of an appropriate subspace based on the Kuhn-Tucker optimality check. Since in our case the objective function is convex and the minimization is performed on a convex set, the Kuhn-Tucker optimality conditions are both necessary and sufficient.

3. Experiments

Several numerical experiments of zig detection were conducted. On Fig. 2-5, the solid lines represent the idealized paths and the dotted lines the actual ("noisy") paths. In the first example (Fig. 2, a-b), the capability to discriminate in the zig direction (right or left maneuver) is demonstrated for two idealized paths. In Fig. 3, a-c, the ability to detect a maneuver in the presence of noise (gaussian) is examined while increasing standard deviation σ. In the third experiment (Fig. 4, a-c), the sensitivity of maneuver detection to course changes of different magnitudes is studied. The last experiment (Fig. 5, a-c) shows the capability of the method to detect a zig maneuver while decreasing the size of the data set n. It can be observed that two measurements after the maneuver were sufficient to detect the maneuver.

On the curvature graphs, a number of "false alarms" appear. They can be discriminated from the actual maneuvers by their respective magnitude levels.

4. Discussion

The distribution of the eigenvalues of the normal matrix $K^{T}K$ serves as a good measure of its ill-conditioning and, indirectly, of the degree of ill-posedness of the inverse problem (1.1). Evaluations yield the ratio of the smallest eigenvalue to the biggest one for n = m = 200 to be in the area of the minimum computer roundoff in double precision. Therefore, since all the pieces of available information are linearly independent, the problem turns out to be of full intrinsic rank [7] and the recovery of the second derivative can be classified as a mildly ill-posed problem. It is recommended in [7] to solve mildly ill-posed problems by standard regularization techniques, which are reduced to imposing smoothness conditions.

If the constraints of the statistical smoothness alone are imposed on (2.1), the equation

$$(K^{T}K + \sigma^{2}C^{-1})\underline{z} = K^{T}\tilde{\underline{u}} , \quad (2.16)$$

i.e. the Euler-Lagrange equation to the variational problem (2.9), will yield the regularized solution. The presence of the positive definite operator $\sigma^{2}C^{-1}$ makes the matrix in the left-hand side of (2.16) positive definite, and thus the variational problem is turned into a well-posed one. Numerically, it means that all the eigenvalues λ_i of the matrix $K^{T}K$ are shifted to safely positive values [6]. This shift represents the effect of imposing the condition of statistical smoothness, which is a statistical equivalent to the Tikhonov regularization [2].

Although the generalized cross-validation technique suggested in [7] compares favorably to the Bayesian estimation technique in that it does not require a priori knowledge of the noise variance, it is, essentially, an eigenvalue shifting technique, like other techniques utilizing constraints of statistical smoothness. In the presence of heavy noise, however, the statistical smoothness technique results in the loss of the compo-

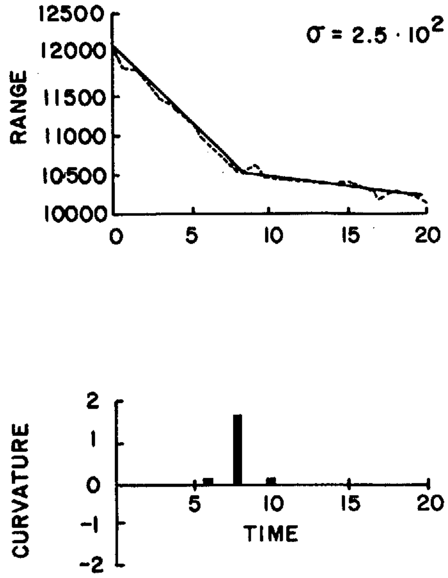

Fig. 2a

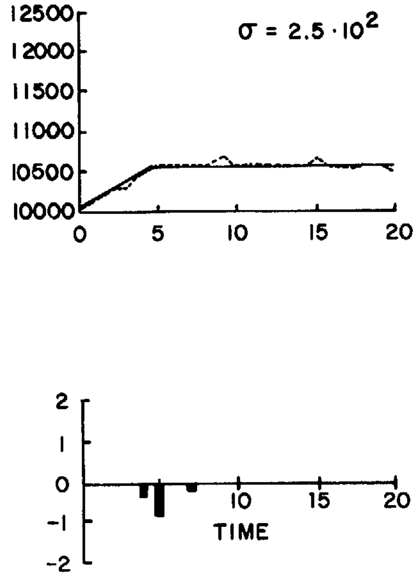

Fig. 2b

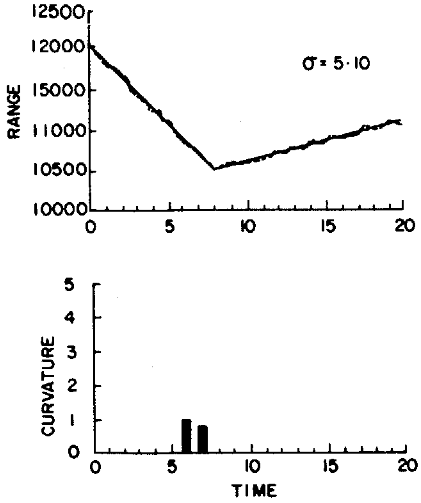

Fig. 3a

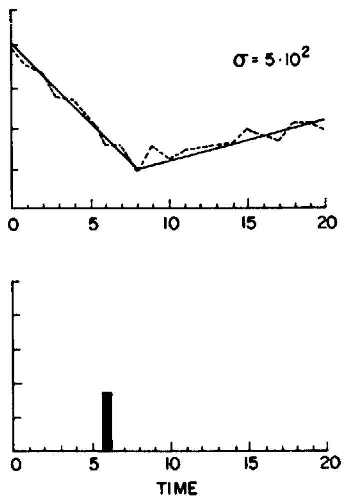

Fig. 3b

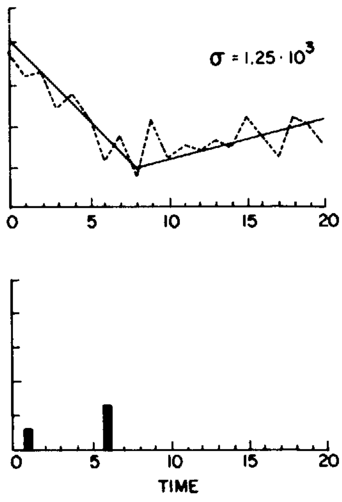

Fig. 3c

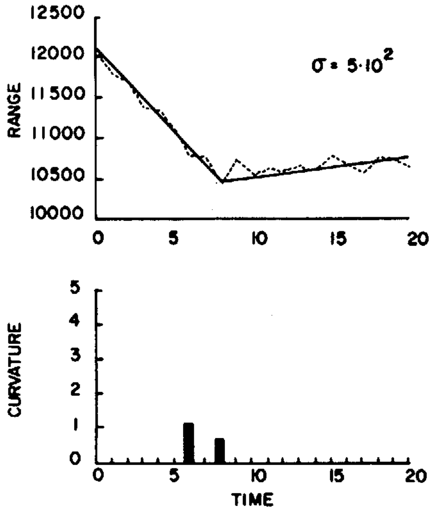

Fig. 4a

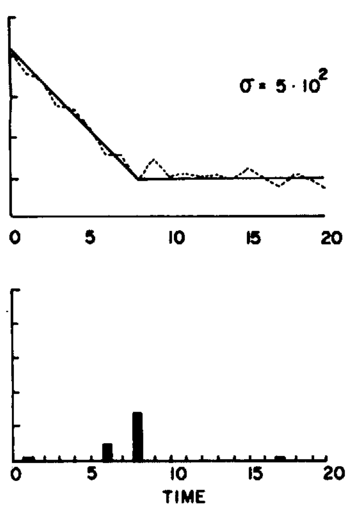

Fig. 4b

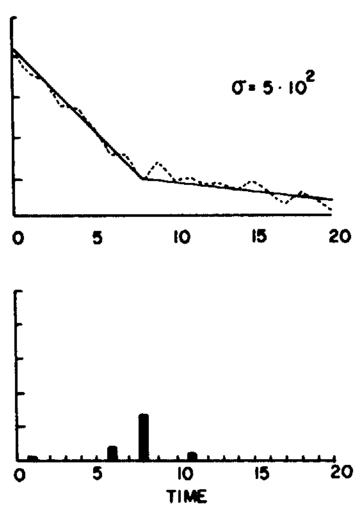

Fig. 4c

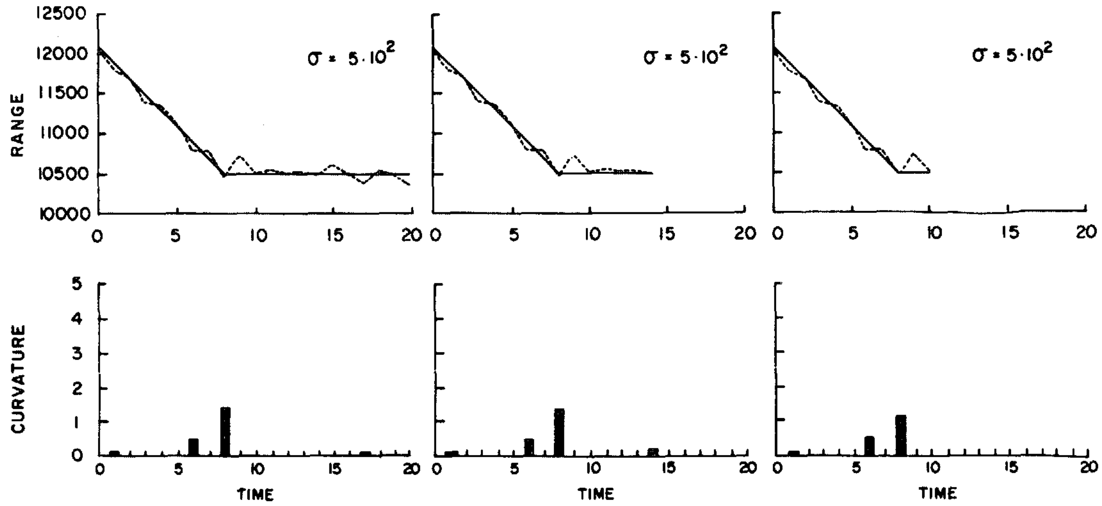

Fig. 5a Fig. 5b Fig. 5c

nents with small eigenvalues. Noise considerably reduces the effective rank of the problem. Experiments show that with the values of the standard deviation σ of the noise as in the examples above, the classical techniques for the recovery of the second derivative, including Tichonov's regularization, fail to detect a maneuver with any reliability. The non-negativity conditions, by contrast, have a strong regularizing effect, which can be explained as "disbalancing" the high-frequency noise components in the solution and therefore suppressing them in the integrand of (1.2). This is achieved without overdamping the solution and yields fairly reliable results in the presence of heavy noise. Experiments show that in this case the solution becomes to a large degree insensitive to the incompleteness of or errors in the statistical data on the noise and the signal to be recovered.

Some other types of maneuver may be covered by an appropriate extension of this technique. The cases (b) and (c) in Fig. 1 can be reduced to the presented technique by replacing the operator of double integration in (1.1) by triple integration. Some other types of maneuvers would be more difficult to reduce to the type considered here.

5. Acknowledgements

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6. References

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