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COMPUTER - AIDED DESIGN OF TWO - DIMENSIONAL INFINITE IMPULSE RESPONSE HALF-PLANE DIGITAL FILTERS WITH OCTAGONAL SYMMETRY

by

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RESUME

Cette publication examine le dessin des filtres digitaux de deux dimensions et d'une résponse d'impulsion infinie de démi-plan, qui possédent une symétrie octagonale. Nous commençons par une structure sérielle et nous réduissons le nombre des paramétres du filtre en imposant la condition de la symétrie octagonale. Par conséquent nous choisissons les sections du filtre de maniére que le filtre qui résulte posséde un dénominateur non-separable qui, néanmoins, peut être controlé facilement pour stabilité en controlant une simple inequalité. Ainsi, les coefficients sont optimisés par l'utilisation de la technique non-linéaire d'optimisation de Fletcher-Powell. Les filtres qui résultent possédent des réponses de fréquense qui ont une bonne symétrie circulaire et des bands étroits de transition.

SUMMARY

This paper considers the design of 2-D IIR half - plane digital filters, which possesses octagonal symmetry. We start from a cascade structure and by imposing the condition of octagonal symmetry we reduce the number of filter parameters. We subsequently choose the filter sections in such a way, that the resulting filter possesses a non-separable denominator, which however can be easily checked for stability, by chequing a simple inequality. The coefficients are then optimized by using the Fletcher - Powell non - linear optimization technique. The resulting filters possess frequency responces which have good circular symmetry and narrow transition bands.

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I. INTRODUCTION

During the past few years there has been a growing interest for the design of 2-D digital filters. Among the different approaches used, computer-aided optimization techniques [1] - [5] have proven efficient and useful. However the major difficulty with these approaches has been the ability to check and ensure the stability of the resulting 2-D transfer function.

In a variety of applications some kind of symmetry in the frequency response is desired for the filter designed. Specifically circular symmetry is very desirable in applications of image precessing and for this reason different methods for the design of 2-D recursive filters which approximate this kind of symmetry, have been proposed [6] - [10]. In [11] it was shown that the circular symmetry can be achieved by a digital transfer function only approximately. Rajan and Swamy [9], [12] - [15] developed the constraints on 2-D transfer functions to have quandrantal, diagonal and octagonal symmetries, as well as the corresponding stability conditions to be satisfied. Furthermore the symmetries impose constraints on the filter coefficients, which result in a significant reduction of the independent parameters for the optimization.

The above mentioned references for the design concern quarter-plane transfer functions. As it is remarked in [16] the quarter-plane design approaches will not yield approximates which are arbitrarily close to a prescribed specification by increasing the order of the filter. This is one of the reasons for the recent attention on the design of half-plane filters, with approaches mainly based on spectral factorization [16] -[17].

In this paper we introduce a design technique for recursive 2-D half-plane filters with octagonal and approximately circular symmetry, under the constraint that the resulting filter be stable. To this end, by applying the symmetry constraints for octagonal symmetry given in [14] for a simple non-trivial case, we obtain a 2-D rational transfer function of low order. The overall transfer function consists of terms of the above form connected in cascade.

Causal stable rational functions designed to date to satisfy octagonal and circular symmetry must have the denominator separable [9], [10], [12], [13]. Furthermore low-pass transfer functions with separable numerators, designed by optimization techniques in [2] & [19] to approximate circularly symmetric frequency responses, turned out to also have separable denominators. The advantage of these designs is that the stability testing reduces to that of checking the stability of 1-D polynomials, which is considerably simpler. However, transfer functions which are separable or have separable denominators cannot be used to design filters possessing sharp cutoff characteristics [13]. The half-plane filters presented here are not required to possess separable denominators, however their stability can be ensured by only checking the relation of two parameters of the denominator polynomial of the transfer function of each term.

II. DEFINITIONS AND NOTATION

We consider a general 2-D infinite impulse response (IIR) transfer function represented by

$$H(z_1,z_2) = \frac{P(z_1,z_2)}{Q(z_1,z_2)} \tag{1}$$

where P, Q are polynomials in z_1, z_2. The frequency response of $H(z_1,z_2)$ is obtained by substituting $z_1=e^{-jw_1}$, $z_2=e^{-jw_2}$, where w_1, w_2 are the normalized frequencies in radians. In this paper we follow the notation of [14].

The magnitude-squared function of (1) is given by

$$F(w_1,w_2) = H(e^{-jw_1}, e^{-jw_2})H^*(e^{jw_1}, e^{jw_2}) \tag{2}$$

where H^* is the transfer function $H(z_1,z_2)$ with complex conjugate coefficients. Let W the 2-D frequency plane $\{(w_1,w_2)/|w_1|\le \pi, |w_2|\le \pi\}$.

Definition 1. Octagonal Symmetry: $F(w_1,w_2)$ is said to possess octagonal symmetry, if $F(w_1,w_2)$ possesses quandrantal symmetry and symmetry about diagonals simultaneously [14], i.e.

$$\begin{aligned} H(z_1,z_2)H(z_1^{-1},z_2^{-1}) &\equiv H(z_1^{-1},z_2)H(z_1,z_2^{-1}) \\ &\equiv H(z_2,z_1)H(z_2^{-1},z_1^{-1}) \end{aligned} \tag{3}$$

Definition 2. Circular Symmetry: $F(w_1,w_2)$ is said to possess circular symmetry if it is invariant on any circular path around the origin in the W plane. In this case $F(w_1,w_2)$ must be of the form [13]

$$F(w_1,w_2)=F_s(w_1^2+w_2^2) \tag{4}$$

where $F_s(.)$ is a single variable function.

We consider here the half-plane filters of the type S_{++} (one of the eight standard types introduced by Ekstorm and Woods [16]. Similar constraints and designs can be easily derived for the other types.

An S_{++} type half half-plane filter can be described as [14]

$$H(z_1,z_2)=\frac{A(z_1,z_2)}{B(z_1,z_2)} \tag{5}$$

where

$$A(z_1,z_2)=a_{oo}+\sum_{m=1}^{M_a} a_{mo}z_1^m + \sum_{m=-M_a}^{M_a}\sum_{n=1}^{N_a} a_{mn}z_1^m z_2^n$$

and

$$B(z_1,z_2)=b_{oo}+\sum_{m=1}^{M_b} b_{mo}z_1^m + \sum_{m=-M_b}^{M_b}\sum_{n=1}^{N_b} b_{mn}z_1^m z_2^n$$

For the above half-plane filter, it was shown in [13] that the following conditions are sufficient to ensure stability:

1) $z_1^{M_a}B(z_1,z_2)$ is a mix-min type polynomial, i.e. none of its singularities or zeros lies on $\{|z_1|=1, |z_2|\le 1\}$,

2) $B(z_1,0)$ is a minimum phase polynomial in z_1.

We now define P, Q which are polynomials in z_1, z_2 as follows:

$$\begin{aligned} P(z_1,z_2) &\triangleq z_1^{M_a}A(z_1,z_2) \\ Q(z_1,z_2) &\triangleq z_1^{M_b} B(z_1,z_2) \end{aligned} \tag{6}$$

III. OCTAGONAL SYMMETRY OF THE TRANSFER FUNCTION

It has been shown in [14], [15], that the numerator of the form (6) should to satisfy the octagonal symmetry constraints (3) and the stability constraints.

A. Numerator. The numerator polynomial $A(z_1,z_2)$ an S_{++} type half-plane filter possesses octagonal symmetry in its magnitude response if and only if, the polynomial $P(z_1,z_2)$, defined in (6), can be written as

$$P(z_1,z_2) = K\prod_{i=1}^{N} P_i(z_1,z_2) \tag{7}$$

where

$$P_i(z_1,z_2),\ i = 1,2,\dots,N$$

are irreducible factors of $P(z_1,z_2)$, such that for each i, $1 \le i \le N$, there are unique j and k (i may be equal to j and/or k), for which

$$P_i(z_1,z_2) \equiv P_j(z_2,z_1) \quad \text{or} \quad P_j(z_2^{-1},z_1^{-1}) z_2^{\alpha_j} z_1^{\beta_j} \tag{8}$$

and

$$P_i(z_1,z_2) \equiv P_k(z_1^{-1},z_2) z_1^{\alpha_k} \quad \text{or} \quad P_k(z_1,z_2^{-1}) z_2^{\beta_k} \tag{9}$$

where

α_i = order of $P_i(z_1,z_2)$ w.r.t. z_1

β_i = order of $P_i(z_1,z_2)$ w.r.t. z_2

The various factors that could be present in $P(z_1,z_2)$ satisfying the above requirments are given in [15].

B. Denominator. The denominator polynomial $B(z_1,z_2)$ of an S_{++} type half-plane filter possess octagonal symmetry in its magnitude response, if and only if, the polynomial $Q(z_1,z_2)$ defined in (6), can be written as

$$Q(z_1,z_2) = Q_{1(+0)}(z_1) \cdot Q_{1(0+)}(z_2) \cdot Q_{3(++)}(z_1,z_2) \cdot Q_{3(-+)}(z_1^{-1},z_2) z_1^{\alpha} \tag{10}$$

where

$$Q_3(z_1,z_2) \equiv Q_3(z_2,z_1) \tag{11}$$

$$Q_3(z_1,0) \equiv Q_3(0,z_2) = \text{constant} \tag{12}$$

and α = order of $Q_3(z_1,z_2)$ w.r.t. z_1

The notation $Q_{(i,j)}$ means that $Q(z_1,z_2)$ is free of zeros in Z_{ij} where [14]

$$Z_{ij} \triangleq \{(z_1,z_2)/z_1 \in R_{1i} \text{ and } z_2 \in R_{2j}\} \text{ for } i,j \ \{+,-,*,0\}$$

and

$$R_{k+} = \{z_k/|z_k| \le 1\} \qquad R_{k-} = \{z_k/|z_k| \ge 1\}$$

$$R_{k*} = \{z_k/|z_k| = 1\} \qquad R_{k0} = \{z_k/|z_k| \ge 0\},\ k=1,2$$

Usually we identify the subscripts "+" with min, "-" with max and "*" with mix. Using this notation $P_{(++)}$ will be called a min-min phase polynomial, $P_{(*,-)}$ a mix-max phase polynomial, $P_{(+0)}$ a minimum phase polynomial in z_1 and so on.

IV. DESIGN OF RECURSIVE FILTERS WITH OCTAGONAL SYMMETRY

A. Choice of Design Structure

We now choose the filter transfer function to be of the following cascade form for several reasons [1].

$$H(z_1,z_2) = A \prod_{\ell=1}^{L} \frac{P^{(\ell)}(z_1,z_2)}{Q^{(\ell)}(z_1,z_2)} \tag{13}$$

where L is the number of sections in cascade and A is a positive gain constant.

In the following we will consider the $P^{(\ell)}(z_1,z_2)$, $Q^{(\ell)}(z_1,z_2)$ to have the form of $A(z_1,z_2)$, $B(z_1,z_2)$ defined in (5). Namely, $A(z_1,z_2)$, $B(z_1,z_2)$ are no polynomials because include negative powers of z_1. Therefore the factors of the form z_1^{α}, z_1^{β} will not be incorporated.

The particular choise of $P^{(\ell)}(z_1,z_2)$, $Q^{(\ell)}(z_1,z_2)$ satisfying the requirements given in Section III is now made in order to obtain one of the simpler but non-trivial form for (13). To this end we choose

$$P^{(\ell)}(z_1,z_2) = a_1^{(\ell)} \left[(z_1+z_1^{-1})+(z_2+z_2^{-1})\right] + a_2^{(\ell)}\left[(z_1+z_1^{-1})(z_2+z_2^{-1})+1\right] \cdot z_2 \tag{14}$$

which satisfies the condition (8).

We also choose

$$Q_{1(+0)}^{(\ell)}(z_1) = Q_{1(0+)}^{(\ell)}(z_2) = 1 \tag{15a}$$

$$Q_{2(++)}^{(\ell)}(z_1,z_2) = b_1^{(\ell)} z_1 z_2 + b_2^{(\ell)} \tag{15b}$$

and substitute to (10) to obtain

$$Q^{(\ell)}(z_1,z_2) = \left[b_1^{\ell} z_1 z_2 + b_2^{(\ell)}\right] \left[b_1^{(\ell)} z_1^{-1} z_2 + b_2^{(\ell)}\right] \tag{16}$$

B. The Optimization Algorithm

The optimization algorithm used, is the Fletcher-Powell non-linear optimization [20] which is formulated as an ℓ_p design technique. In this paper we use the mean square error criterion, i.e. p=2. Let the magnitude characteristic $Y = [Y_{mn}]$ defined on the discrete set of frequency pairs $(w_1^{(m)}, w_2^{(n)})$, $m = 1,\dots, M$; $n = 1,\dots,N$, belong in a region S of the plane (w_1,w_2). The region S is considered to be the triangle whose vertices are the points $(0,0)$, $(\pi,0)$, $(0,\pi)$.

Let the

$$\Phi = [\Phi_1^T, \Phi_2^T] \tag{17}$$

where

$$\Phi_1^T = [a_1^{(1)}, a_2^{(1)}, \dots, a_1^{(\ell)}, a_2^{(\ell)}, \dots, a_1^{(L)}, a_2^{(L)}]$$

$$\Phi_2^T = [b_1^{(1)}, b_2^{(1)}, \dots, b_1^{(\ell)}, b_2^{(\ell)}, \dots, b_1^{(L)}, b_2^{(L)}]$$

and the $a_i^{(\ell)}$, $b_i^{(\ell)}$, $i = 1,2$, $\ell = 1,\dots, L$ are the explicit parameters of the transfer function. The dimension of the vector Φ is (Tx1), where T=4L.

The performance index is given by

$$J(\Phi) = \sum_{m=1}^{M} \sum_{n=1}^{N} \left[|H_{mn}| - Y_{mn}\right]^2 \tag{18}$$

where

$$H_{mn} = H(z_{1m}, z_{2n}) = A \prod_{\ell=1}^{L} \frac{P^{(\ell)}(z_{1m},z_{2n})}{Q^{(\ell)}(z_{1m},z_{2n})} = AF_{mn}$$

and

$$(z_{1m},z_{2n}) = (e^{-jw_1^{(m)}}, e^{-jw_2^{(n)}})$$

The problem is to select the elements of the vector Φ that minimize the performance index $J(\Phi)$. Stability is considered later. The coefficient A (which is the only linear parameter of the objective function) can be optimized separately, by differentiating (18) with respect to A, by the relation

$$A_{opt} = \frac{\sum_{m=1}^{M}\sum_{n=1}^{N} Y_{mn}\,|F_{mn}|}{\sum_{m=1}^{M}\sum_{n=1}^{N} |F_{mn}|^2} \qquad (19)$$

The optimization algorithms calls for the computation of the gradient vector ∇J. To reduce the quantization error in the calculations of ∇J we choose an analytical approach in the computation. To this end, using (13),(18) we obtain

$$\frac{\partial J(\Phi)}{\partial \varphi_i} = 2 \sum_{m=1}^{M}\sum_{n=1}^{N} \left[[A_{opt}|F_{mn}| - Y_{mn}] \cdot \frac{\partial}{\partial \varphi_i} [A_{opt}|F_{mn}|] \right] \qquad (20)$$

The derivative term is simply

$$\frac{\partial}{\partial \varphi_i}[A_{opt} \cdot |F_{mn}|] = A_{opt} \frac{\partial |F_{mn}|}{\partial \varphi_i}$$

$$= A_{opt} \cdot \frac{1}{|F_{mn}|} \mathrm{Re}\left[F_{mn}^{*} \cdot \frac{\partial F_{mn}}{\partial \varphi_i}\right]$$

$$= A_{opt}|F_{mn}|\,\mathrm{Re}\,[X_i(m,n)] \qquad (21)$$

since $\partial A_{opt}/\partial \varphi_i$ is zero when assumed over the grid $(w_1^{(m)}, w_2^{(n)})$, m = 1,..., M, n=1,...,N, already defined as its optimum value.

The quantities $X_i(m,n)$, i=i,...,T are the elements of the vector $\underline{X}$, defined by

$$\underline{X} = [x_1^1, x_2^1, x_3^1, x_4^1, \ldots, x_1^{(\ell)}, x_2^{(\ell)}, x_3^{(\ell)}, x_4^{(\ell)}, \ldots, x_4^{(L)}]$$

where

$$x_1^{(\ell)} = \frac{z_{1m}+z_{1m}^{-1}+z_{2n}+z_{2n}^{-1}}{P^{(\ell)}(z_{1m},z_{2n})}$$

$$x_2^{(\ell)} = \frac{(z_{1m}+z_{1m}^{-1})(z_{2n}+z_{2n}^{-1})+1}{P^{(\ell)}(z_{1m},z_{2n})}$$

$$x_3^{(\ell)} = \frac{2b_1 z_{2n}^2 + b_2(z_{1m}+z_{1m}^{-1})}{Q^{(\ell)}(z_{1m},z_{2n})}$$

$$x_4^{(\ell)} = \frac{b_1(z_{1m}+z_{1m}^{-1})z_{2n}+2b_2}{Q^{(\ell)}(z_{1m},z_{2n})}$$

and

m = 1,2,..., M, n = 1,2,...,N

C. Stability Test and Stabilization

The denominator (16) of the S_{++} type half-plane filter is stable if the polynomial of each section ℓ,

$$Q_{3(++)}^{(\ell)}(z_1,z_2) = b_1^{(\ell)} z_1 z_2 + b_2^{(\ell)}, \quad \ell = 1,2,\ldots,L$$

is free of zeros in the region $Z_{++} = \{(z_1,z_2)/|z_1| \leq 1, |z_2| \leq 1\}$. It is readily seen that a sufficient condition for the satisfaction of the previous condition is

$$|b_2^{(\ell)}| > |b_1^{(\ell)}| \qquad (22)$$

Consequently the stability test is very easy. In each final step of the optimization procedure, (22) is tested. If it is not satisfied, $|b_1^{(\ell)}|$ is reduced (or $|b_2^{(\ell)}|$ is increased) and the optimization is repeated. This approach simplifies the problem of checking the stability, which is a complex problem for the case of nonseparable denominators.

V. EXAMPLES

Example 1. One section low-pass filter.

The specifications here were passband and stopband radii of 0.1π and 0.3π respectively. The maximum ripples in the passband and stopband regions are 0.15 and 0.3 respectively.

134 grid points are chosen on the region S at the intersections of 34 circles with radial lines.

In the stopband, transition and stopband regions the number of radii are taken to be 12,10,11 and the number of equidistant points per radius are taken to be 5, 3,4 respectively.

The values of the parameters a_1, a_2, b_1, b_2, A are given in Table I. In order to obtain a picture of the magnitude response, the desired values and the corresponding values of the designed filter are given in Table II for a number of radii.

Table I

CONSTANT A	a_1	a_2	b_1	b_2
0.454794	2.05812	1.73565	3.80538	-6.39386

Table II

Radius of circle	Desired value	Corresponding values
0.008π	1.0	1.13
0.1π	1.0	0.82
0.2π	0.5	0.46
0.3π	0.0	0.3
0.5π	0.0	0.2
1.0π	0.0	0.015

Example 2. Two sections lowpass filter.

The specifications are similar to Ex. 1. The maximum ripples in the passband and stopband region are 0.096 and 0.175 respectively.

The values of the estimated parameters are given in Table III and values of the magnitude response in Table IV.

Table III

CONSTANT A	SECTION k	a_1^k	a_2^k	b_1^k	b_2^k
2.2192	1	2.02749	1.77205	2.51682	-7.45759
	2	2.02749	1.77140	2.51511	-7.4533

Table IV

Radius of Circle	Desired value	Corresponding values
0.0017π	1.0	1.07
0.1π	1.0	0.904
0.2π	0.5	0.53
0.3π	0.0	0.175
0.5π	0.0	0.023
1.0π	0.0	0.002

Example 3 : Three sections lowpass filter.

The specifications are similar to Ex. 2. The maximum ripples in the passband and stopband regions are 0.08 and 0.097 respectively.

The values of the estimated parameters are given in table V and values of the magnitude response in Table VI.

Table V

CONSTANT A	SECTION k	a_1^k	a_2^k	b_1^k	b_2^k
	1	3.11691	2.58815	1.36450	-16.05326
7920.72	2	-2.38948	2.57605	1.22773	-15.53927
	3	2.38864	1.48668	1.50611	-17.060

Table VI

Radius of circle	Desired value	Corresponding values
0.0017π	1.0	1.06
0.1π	1.0	0.92
0.2π	0.5	0.55
0.3π	0.0	0.097
0.5π	0.0	0.0076
1.0π	0.0	0.026

In all the above cases, very good circular symmetry has been attained, since the values of the magnitude response in the grid points of each radius are almost the same.

VI . CONCLUDING REMARKS

1) In this paper results are presented for computer-aided design of 2-DIIR half-plane digital filters with octagonal symmetry. There is great possibility of choise of the structure of the rational function, satisfying the symmetry and stability constraints. The form of the function used simplifies the optimization procedure. However more complex and higher order functions have more possibilities since they possess a greater number of degrees of freedom.

2) The design technique proposed is applied to half-plane filters, which are more efficient than the usually designed quarter-plane filters.

3) The nonseparable denominator used gives the possibility to design filters with sharp cut-off characteristics . Furthermore their stability can be very easily checked by checking the satisfaction of a single inequality.

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