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# DELAY-DOPPLER RADAR-IMAGING USING CHIRP-RATE MODULATION

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## RESUME

On montre l'analogie entre l'emission d'image du radar doppler avec retard de temps et la tomographie utilisant l'emission du positon. Ceci suggere de nouveaux logarithmes de traitement d'informations pour le probleme d'emission d'image par radar, ce qui peut permettre une meilleure visualisation des objectifs. On considere que la structure du recepteur costituee d'un filtre adapte a bande passante d'un detecteur d'enveloppe quadratique et d'un traitement d'informations specialise peut produire des images. On peut utiliser des signaux de radar ayant des structures classiques de lobe lateral ainsique autres caracteristiques. Le traitement specialise necessaire est couteux mais l'architecture du logarithme peut permettre des implementations en temps reel.

## SUMMARY

An analogy is identified between imaging in delay-doppler radar and positron-emission tomography. This suggests new processing algorithms for the radar-imaging problem that may permit improved visualization of targets. A receiver structure consisting of a bandpass matched-filter, square-law envelope-detector, and specialized processing is proposed to produce images. Radar signals having practical sidelobe structures and other features can be accommodated. The specialized processing required is demanding, but the architecture of the algorithm may permit real-time implementations.

## I. Introduction

An analogy has recently become recognized between delay-doppler imaging- radar systems and tomographic systems used in clinical radiology. It is an analogy holding the possibility of improving radar imaging because the use of matched filtering for noise suppression is suggested even by initial comparisons, and, more importantly, because it suggests a line of thinking by which new mathematical models for the radar-imaging problem might be formulated and solved for improved processing that accounts for dominant effects including noise. This observation was made by M. Bernfeld [1], and it also appears in a different form in the work of D. Mensa, S. Halevy, and G. Wade [2]. These studies all draw the analogy to a tomography system wherein the data available for processing are in the form of idealized, noise-free line-integrals through the object being imaged, a situation that is well approximated with x-ray tomography systems because x-ray sources can be highly collimated so as to form narrow x-ray beams of high intensity that are passed through the object being imaged. The analogy that has been articulated in [1] and [2] is not a particularly good one because the ambiguity functions of radar signals of interest (for example, the frequency-stepped chirp-signals of M. Prickett, D. Wehner, and C. Chen [3] and D. Wehner [4]) have sidelobe structures and other features that cause a departure from idealized line-integrals and also because noise can be nonnegligible in some radar-imaging situations. The purpose of this paper is to suggest an approach which may permit the removal of the restriction of noise-free line-integrals so that general ambiguity-functions can be accommodated and to suggest the development of mathematical models so that the effects of noise can be recognized in the processing of radar returns. This will be accomplished by making an analogy to newer tomography systems that employ positron-emitting radionuclides rather than x-ray sources.

## II. Background and Concept Outline

<u>radar imaging</u>. Suppose that $\sigma(\tau,f)$ is the target scattering-function [5, p. 448], which is the average reflectivity as a function of delay $\tau$ and doppler $f$. Also, let $a(\tau,f)$ denote the ambiguity function of the transmitted radar-signal [5, p. 279]. Then, in the absence of noise, the output $p(\tau,f)$ of a radar receiver consisting of a bandpass matched-filter (BPMF, matched to the transmitted radar signal) followed by a square-law envelope-detector (SLED) is the convolution of the target scattering-function and the ambiguity function of the transmitted signal [5, pp. 462-463],

$$p(\tau,f) = \iint \sigma(\tau',f')a(\tau-\tau',f-f')d\tau'df'. \tag{1}$$

For the delay-doppler radar-imaging problem without noise, we consider a sequence of target illuminations by chirp-FM signals, with each signal having a different chirp rate. The effect of changing the chirp rate of a signal on its ambiguity function is to rotate the ambiguity function to an angle, say $\theta$, in the delay-doppler plane [5, p. 291]. To indicate this dependency, we change the notation in (1) to

$$p_\theta(\tau,f) = \iint \sigma(\tau',f')a_\theta(\tau-\tau',f-f')d\tau'df', \tag{2}$$

where $\theta$ is determined by the chirp rate relative to the radar pulse without chirp-FM. The noise-free radar-imaging problem is to observe the output of the BPMF-SLED receiver $p_\theta(\tau,f)$ for a sequence of target illuminations having different chirp-FM rates $\theta = \theta_0$, $\theta_1$, ... , $\theta_N$ and to determine the scattering function $\sigma(\tau,f)$.

<u>tomographic imaging</u>. We now turn to recent developments in positron-emission tomographic-imaging systems where a relation analogous to (2) occurs. In these systems, a positron-emitting radionuclide is introduced inside a patient, and the resulting activity is observed externally with an array of scintillation-detectors that surround the patient in a planar, ring geometry. When a positron is produced in a radioactive decay, it annihilates with an electron producing two high energy (512 kev) photons that propagate in opposite directions along a line. In the first systems employing positron emission, the line-of-flight of the two oppositely propagating photons is sensed for each detected event; these data are then organized according to their propagation angle and processed with the same algorithms used in x-ray tomography. The result is an estimate of the two-dimensional spatial distribution of the radionuclide within the patient in the plane of the detector ring. Recent developments relevant to this paper have resulted from improvements in high speed electronics and detector technology, which make it feasible to measure with useful accuracy not only the line-of-flight of annihilation photons but also their differential time-of-flight. The result is that, in the absence of noise, the measurements are in the form of (2) with $\sigma(\tau,f)$ being a two-dimensional activity distribution to be imaged, and with $a_\theta(\tau,f)$ being the error density associated with measuring the location of an annihilation event [6].* The noise-free imaging problem of emission tomography is to observe the line-of-flight and the time-of-flight of the sequence of detected annihilation photons, modeled on the average by $p_\theta(\tau,f)$ in (2), and to determine the two-dimensional activity distribution $\sigma(\tau,f)$. Here, $a_\theta(\tau,f)$ is a known function determined by instrumentation errors, and $p_\theta(\tau,f)$ is the number of detected events having a line-of-flight with angle $\theta$ and differential time-of-flight corresponding to position $(\tau,f)$ along the line-of-flight. Data quantized to ninty-six angles ($\theta_i = 180i/96$, $i = 0, 1, \ldots, 95$) and to 128-by-128 positions are collected in the instrument being developed at Washington University [7]. The error density $a_\theta(\tau,f)$ is determined by both the physical size of the crystals used in the scintillation detectors (resulting in about a 1 cm uncertainty transverse to the line-of-flight) and the timing resolution of the electronic circuitry used to measure the differential propagation-time (resulting in about a 7 cm spatial uncertainty along the line-of-flight). For present systems, this density is reasonably modeled by a two-dimensional, elliptically asymmetric Gaussian-function having its major axis oriented with the line-of-flight and its minor axis oriented transversally to this. For the radar-imaging problem, this density corresponds to the ambiguity function of a radar pulse having an envelope that is a Gaussian function and a phase that is a linear-FM chirp.

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* $\sigma(\tau,f)$, $a_\theta(\tau,f)$, and $p_\theta(\tau,f)$ correspond to $\lambda(x)$, $p_\varepsilon(x|\theta)$, and $\mu(u,\theta)$, respectively in [6], where x and u are two-dimensional vectors.

In summary:

a. For delay-doppler radar-imaging, we suppose that the target is illuminated by a sequence of radar pulses each having a distinct FM-chirp rate corresponding to angles $\theta = \theta_0, \theta_1, \ldots, \theta_N$ spanning the range from 0 to 180°. A BPMF-SLED receiver produces data $p_\theta(\tau,f)$ for $\theta_0, \theta_1, \ldots, \theta_N$ and quantized values of $(\tau,f)$. The ambiguity function $a_\theta(\tau,f)$ is known. The problem is to estimate the target scattering function $\sigma(\tau,f)$ using the relationship in (2).

b. For emission-tomography imaging when both time-of-flight and line-of-flight information is available, we have event data $p_\theta(\tau,f)$ at angles $\theta = \theta_0, \theta_1, \ldots, \theta_N$ spanning 0 to 180° and quantized values of $(\tau,f)$. The measurement-error density $a_\theta(\tau,f)$ is known. The problem is to estimate the activity distribution $\sigma(\tau,f)$ using the relationship in (2).

Substantial progress has recently been made toward solving the tomography-imaging problem. We next explore the implications of this for radar imaging.

## III. Preliminary Considerations

In this section, we briefly outline what the results in [6], [8], and [9] suggest for the radar-imaging problem. The algorithm for solving (2) that is proposed in [6] and evaluated in [8] and [9] is derived by applying statistical-estimation theory to a mathematical model that accounts for the noise and other effects seen in an emission-tomography system having time-of-flight measurements. This noise is Poisson distributed, as might be expected because of the quantum nature of radioactive decay, an effect well modeled by a Poisson process with intensity $\sigma(\tau,f)$ proportional to the concentration of the radioactive source. It is argued in [6] that the measured data (i.e., line- and time-of-flight of annihilation photons) are also Poisson distributed, with the intensity being $p_\theta(\tau,f)$ in (2). Maximum-likelihood estimation is then used to estimate $\sigma(\tau,f)$. This approach to algorithm development has been extended in [10] for more accurate reconstructions at the expense of greatly increased computation.

For the purpose of this discussion, we now neglect the effects of noise and statistical fluctuations in the measured data and take $p_\theta(\tau,f)$ as the measurement, as we have described in Sec. 2. The algorithm developed in [6] then suggests the following for the radar-imaging problem.

The output $p_\theta(\tau,f)$ of the BPMF-SLED receiver is three dimensional because it is a function of the three independent variables $\theta$, $\tau$, and $f$. The target image sought $\sigma(\tau,f)$ is two dimensional. Thus, a three-dimensional to two-dimensional transformation of $p_\theta(\tau,f)$ is required as part of the processing. This is accomplished in two steps in [6], but the algorithm can be partitioned in other ways too because it is linear. The first step is to form a two-dimensional ''preimage array.'' This is accomplished by convolving the data $p_\theta(\tau,f)$ obtained at each FM-chirp rate $\theta$ with a weighting function $w_\theta(\tau,f)$ and then summing the results over $\theta$; that is, we form the functions

$$f_\theta(\tau,f) = \iint p_\theta(\tau',f')w_\theta(\tau-\tau',f-f')d\tau'df', \tag{3}$$

and then we obtain the two-dimensional preimage $f(\tau,f)$ according to

$$f(\tau,f) = \int_0^\pi f_\theta(\tau,f)d\theta. \tag{4}$$

The formation of this preimage corresponds to some extent with back-projection step of the 'unfiltered back-projection, post two-dimensional filtering' approach to idealized line-integral tomography. Examples of weighting functions that might be adopted are indicated as follows.

<u>example 1</u>:

$$w_\theta(\tau,f) = (\delta\tau\ \delta f)^{-1} I_{\delta\tau}(\tau) I_{\delta f}(f),$$

where

$$I_{\delta\tau}(\tau)I_{\delta f}(f) = 1, \quad |\tau| \le \delta\tau/2,\ |f| \le \delta f/2$$
$$= 0, \quad \text{otherwise}.$$

Here, $w_\theta(\tau,f)$ is unity for delays and dopplers in a small bin located at $\tau$ and $f$ in the delay-doppler plane and is zero otherwise, independently of the sweep rate $\theta$. In this case, $f_\theta(\tau,f)$ equals $p_\theta(\tau,f)$, and the preimage is

$$f(\tau,f) = \int_0^\pi p_\theta(\tau,f)d\theta.$$

This choice of $w_\theta(\tau,f)$ might be reasonable if the ambiguity function $a_\theta(\tau,f)$ is concentrated about the origin $(\tau,f) = (0,0)$, which requires a very wideband radar signal. Then, $p_\theta(\tau,f)$ equals $\sigma(\tau,f)$, and the preimage is obtained simply by post detection integration in each delay-doppler bin without further processing.

<u>example 2</u>: Suppose that $w_\theta(\tau,f)$ is unity for values of delay and doppler within a narrow strip of width $\delta$ passing through the origin of the delay-doppler plane at angle $\theta$ and that $w_\theta(\tau,f)$ is zero otherwise (see Fig. 6 in [6]). Then $f_\theta(\tau,f)$ is a strip integral, or line integral for $\delta$ small, through the data $p_\theta(\tau,f)$, which corresponds to unfiltered back-projection in tomography. This is similar to the situation considered by M. Bernfeld [1].

Our experience in positron-emission tomography suggests using $w_\theta(\tau,f) = a_\theta(\tau,f)$ to form the preimage. This corresponds to taking the value of the BPMF-SLED signal $p_\theta(\tau,f)$ observed at each value of delay and doppler $(\tau,f)$ and distributing it over the delay-doppler plane according to the ambiguity function $a_\theta(\tau,f)$. This approach is the one now used routinely in emission-tomography systems having time-of-flight data; its performance for emission tomography is discussed in [9]. If the mathematical development in [6] carries over to the radar-imaging problem, this choice of weighting function would be motivated by noting that the resulting $f_\theta(\tau,f)$ is the maximum-likelihood estimate of the delay-doppler reflectance in the target that led to the measurement $p_\theta(\tau,f)$ assuming a priori that $\sigma(\tau,f)$ is uniform.

The second processing step is to obtain the target image from the preimage. This is done to within a resolution function $h(\tau,f)$, which defines a ''desired image'' according to

$$d(\tau,f) = \iint h(\tau-\tau',f-f')\sigma(\tau',f')d\tau df.$$

We have found that including such a resolution function is important in processing emission-tomography data as a way to trade off resolution and smoothing for noise suppression. In [6], a narrow, two-dimensional, circularly symmetric Gaussian resolution-filter is used. Let $\hat{d}(\tau,f)$ denote the estimate of $d(\tau,f)$ obtained by processing the preimage $f(\tau,f)$. Also, let $D(u,v)$ and $F(u,v)$ denote the two-dimensional Fourier transforms of $\hat{d}(\tau,f)$ and $f(\tau,f)$, respectively. The from [6, eq'n. 17],

$$D(u,v) = H(u,v)F(u,v)/G(u,v), \tag{5}$$

where $H(u,v)$ is the transform of $h(\tau,f)$ and $G(u,v)$ is the transform of the function $g(\tau,f)$ defined according to

$$g(\tau,f) = (1/\pi)\int_0^\pi a_\theta(\tau,f)w_\theta(\tau,f)d\theta. \tag{6}$$

The image $\hat{d}(\tau,f)$ is obtained from $D(u,v)$ by a two-dimensional, inverse Fourier transformation. The functions $g(\tau,f)$ and $G(u,v)$ are precomputable since they depend only on the ambiguity function and the weighting function used to form the preimage and not on the measured data. For the choice

$$w_\theta(\tau,f) = a_\theta(\tau,f),$$

the function $g(\tau,f)$ is the average over $\theta$ of the square of the ambiguity function. In [6], $a_\theta(\tau,f)$ is a two-dimensional, asymmetric Gaussian function, and $g(\tau,f)$ is a Bessel function. The derivation in [6] does not require that $a_\theta(\tau,f)$ be Gaussian, but $g(\tau,f)$ will usually need to be evaluated numerically for practical ambiguity functions.

The processing we have described for the radar-imaging problem is motivated by the processing derived from a mathematical model for the emission-tomography imaging problem. We are optimistic that improved radar images will result from its use, but further research is needed to explore this.

## IV. Algorithm Architecture

The architecture of the algorithm defined by (3)-(6) is similar to that discussed in [11] for tomographic imaging. Data acquired for each doppler rate can be processed in parallel and then combined to form $f(\tau,f)$ according to (4), and the processing in (3) required for each doppler rate can be pipelined. The processing implemented in current emission-tomographs is performed in the spatial rather than Fourier domain.

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