IDENTIFICATION OF ARMA PROCESSES FROM NOISY OUTPUT

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RESUME

L'identification d'un processus ARMA (auto-regressive-moving average) consiste à déterminer ses ordres et coefficients. Ce papier considère l'identification ARMA utilsant seulement la séquence de sortie, z_u, corrumpue par le bruit. Une difficulté fondamentale est l'estimation de la puissance (véritable) de sortie du processus, R_o, nécessaire pour déterminer l'ordre ainsi que pour estimer les paramètres, et qui ne peut pas être considérée comme la somme des carrés de z_u à cause de la présence du bruit.

Une solution est proposée dans laguelle on calcule d'abord les coefficients $\hat{a}_k$ de AR obtenne à l'aide d'une Equation Yule-Walker d'Ordre Supérieur (Cette équation n'a pas besoin de R_o). Ensuite pour une gamme de R_o expérimentaux, on calcule les groupe correspondents des coefficients $\tilde{a}_k$ de AR à l'aide d'une approximation AR à la methode AR d'estimation ARMA. La R_o qui produit le groupe de $\tilde{a}_k$ les plus proches de $\hat{a}_k$ est choisie comme l'estimée de la puissance de sortie. Des qu'une estimée fiable de R_o est disponible, l'identification peut suivre les techniques standards. L'efficacité de cette méthode est supportée par deux exemples de simulation.

SUMMARY

Identification of an ARMA process consists of determining its orders and coefficients. This paper considers ARMA identification using only the noise corrupted output sequence z_n. A fundamental difficulty is in the estimation of the true process output power R_o, needed both for order determination and parameter estimation, and cannot be simply taken as the sum of the squares of the z_n because they contain noise.

A solution is proposed which first computes the AR coefficients $\hat{a}_k$ from the High Order Yule-Walker Equation (This equation does not require R_o.). Then for a range of test R_o, corresponding sets of AR coefficients $\tilde{a}_k$ are computed via an AR approximation to ARMA estimation method. The R_o that produces the set of $\tilde{a}_k$ closest to the $\hat{a}_k$ is chosen as the estimate of the output power. Once a reliable R_o estimate is available, the identification can follow standard techniques. The effectiveness of the scheme is supported by two simulation examples.

I. INTRODUCTION

System identification has many applications in areas such as control, econometrics and spectral estimation. In the literature [1], identification is usually taken to mean the determination of the orders (i.e., the number of poles and zeros if the system is linear) as well as the parameters (the poles and zeros) of the unknown system. Occasionally, the term identification is incorrectly used to represent parameter estimation of the system coefficients only, which necessarily assumes that the system orders are known a priori. In signal processing, there has been interests in the identification of an autoregressive-moving average (ARMA) process [2-4], because of its relation to speech modeling and spectral estimation.

Some of the early work on ARMA order determination were that of Chow [5] and Akaike, who extended in [6] his Akaike Information Criterion (AIC) for AR processes to ARMA processes. Chow's method uses the relationship between the linear dependency of the output autocorrelation functions and the ARMA orders. This was modified by Chan and Wood [4] who gave a simpler implementation. Akaike's approach requires the selection of many possible ARMA orders, estimation of the corresponding ARMA coefficients, and finally generation of the AIC's. Thus length computations may be required.

When the ARMA output is corrupted by noise, none of the methods above can be carried over for order determination because they require good estimates of the output power (denoted R_o), which is not available when noise is present. Chow [5] did allow additive noise in the output but assumed known noise variance. This paper considers the very general problem of ARMA identification given only white noise corrupted output. The AR order is first determined from the order determination array (ODA) of [4], followed by an estimation of the AR coefficients. Let these be $\hat{a}_k$. As will be seen, this portion of the identification can be accomplished without the knowledge of R_o. Next, the AR and MA coefficients are obtained from the method of Graupe, et al [7]. Here, R_o is needed. Since its direct estimate from the output sequence is not feasible because of noise, a range of trial R_o is taken to compute several sets of AR and MA coefficients. Let these be $\tilde{a}_k$ and $\tilde{b}_i$ respectively. A comparison is then made between the $\hat{a}_k$ and $\tilde{a}_k$. The R_o that produces the $\tilde{a}_k$ closest to the $\hat{a}_k$ is chosen as the estimate of the output power. Once this quantity is found, the MA order and coefficient estimation follow known procedures. Estimating the MA coefficients is a nonlinear problem [10]. However, by approximating an ARMA process as an AR process [7], linear solution is possible. There are three more sections to follow in this paper, beginning with Section II which contains the development of the identification scheme. The simulation results are in Section III, followed by Section IV which gives several variations of the scheme, together with the conclusions.

II. ARMA IDENTIFICATION

Given a finite sequence $\{z_n\}$, n=0,...,N-1 as the sum of the output of an ARMA process plus noise, that is,

$$z_n = x_n + \xi_n \tag{1}$$

where

$$x_n = \sum_{k=1}^{p} a_k x_{n-k} + \sum_{i=o}^{q} b_i W_{n-i}\ , \quad b_o = 1 \tag{2}$$

is the output of an ARMA process, and $\{W_n\}$ and $\{\xi_n\}$ are uncorrelated bandlimited white noise sequences [8], the problem is to determine, from $\{z_n\}$ only, the order (p,q) and coefficients a_k, b_i of the ARMA process.

We first briefly review the results in [4] and [7] since they are central to the present approach. Let

$$R_{xx}(\ell) \triangleq E\{x_{n+\ell}\, x_n\} \tag{3}$$

be the autocorrelation function of x_n and consider the ODA

$$\begin{matrix} R_{xx}(a) & R_{xx}(a-1) \ldots\ldots R_{xx}(0) & R_{xx}(-1) & R_{xx}(-2) \ldots R_{xx}(-b) \\ R_{xx}(a+1) & \ldots\ldots\ldots\ldots R_{xx}(1) & R_{xx}(0) & R_{xx}(-1) \ldots\ldots\ldots \\ \vdots & & & \\ R_{xx}(2a+b) & \ldots\ldots\ldots\ldots\ldots\ldots & & \ldots\ldots R_{xx}(-a) \end{matrix} \tag{4}$$

which has the dimension (a+1+b) × (a+1+b). The integers a and b are chosen to be larger than the maximum expected MA and AR orders, respectively. It was shown in [4] that if the process has orders (p,q), a check for column linear dependency (ℓ.d.) in the ODA (starting from the left, the first column has index zero) will reveal that the column of index p is linearly dependent on the past p columns. Further, on continuing the ℓ.d. check, linear independence will be reintroduced at the column of index m, where m = p-q+a. The above follows from the simple relationships

$$R_{xx}(\ell) = \sum_{k=1}^{p} a_k R_{xx}(\ell-k) \qquad \text{for } \ell > q \tag{5}$$

and

$$R_{xx}(\ell) \neq \sum_{k=1}^{p} a_k R_{xx}(\ell-k) \qquad \text{for } \ell \leq q \tag{6}$$

When the orders (p,q) are known, estimation of the coefficients a_k, b_i follows the method of [7], which approximates an ARMA process by a pure AR process. We give an alternate development of the results in [7] that includes the case of q>p, which was not considered in [7].

The AR approximation of (2) is

$$x_n \simeq \sum_{\ell=1}^{Q} C_\ell\, x_{n-\ell} + W_n \tag{7}$$

or

$$W_n \simeq x_n - \sum_{\ell=1}^{Q} C_\ell\, x_{n-\ell} \tag{8}$$

where C_ℓ, ℓ=1,..,Q are the coefficients of the AR process and Q > p+q is chosen sufficiently large to ensure an accurate representation of (2) by (7) but its exact value is not critical. Thus if Q=20 is sufficient, using Q=18 or 22 will not affect the results. The process (2), however, must be stable and invertible [7]. These C_ℓ coefficients are calculated from

$$C = R^{-1}\, r \tag{9}$$

where

$$C^T = [C_1\ C_2\ \ldots\ C_Q] \tag{10}$$

and

$$R = \begin{bmatrix} R_{xx}(0) & R_{xx}(-1) \ldots R_{xx}(1-Q) \\ R_{xx}(1) & R_{xx}(0)\ \ldots\ldots\ldots \\ \vdots & \\ R_{xx}(Q-1) & \ldots\ldots\ldots R_{xx}(0) \end{bmatrix}, \quad R = \begin{bmatrix} R_{xx}(1) \\ R_{xx}(2) \\ \vdots \\ R_{xx}(Q) \end{bmatrix} \tag{11}$$

To relate the C_ℓ to a_k, b_i, substitute (8) into (2) to give

$$x_n \simeq \sum_{k=1}^{p} a_k x_{n-k} + \sum_{i=0}^{q} b_i [x_{n-i} - \sum_{\ell=1}^{Q} C_\ell\, x_{n-i-\ell}] \tag{12}$$

Equating coefficients of x_n, x_{n-1}, to x_{n-p} results in

$$\begin{bmatrix} a_1 \\ a_2 \\ \vdots \\ a_p \end{bmatrix} = -\begin{bmatrix} b_1 \\ b_2 \\ \vdots \\ b_p \end{bmatrix} + \begin{bmatrix} 1 & 0 \ldots\ldots\ldots 0 \\ b_1 & 1\ \ 0 \ldots\ldots 0 \\ b_2 & b_1\ \ 1\ \ 0 \ldots 0 \\ \vdots & \\ b_{p-1} & b_{p-2} \ldots b_1\ \ 0 \end{bmatrix} \begin{bmatrix} C_1 \\ C_2 \\ \vdots \\ C_p \end{bmatrix} \tag{13}$$

where, if $p>q$, we let $b_{q+1} = b_{q+2} = \dots = b_p = 0$. If $p \geq q$, continuing the process of equating coefficients of $x_{n-p-1}, \dots, x_{n-q-Q}$ gives

$$\begin{bmatrix} c_{p+1} \\ c_{p+2} \\ \vdots \\ c_{p+q} \\ \vdots \\ c_Q \end{bmatrix} = - \begin{bmatrix} c_p & c_{p-1} \cdots\cdots & c_{p-q+1} \\ c_{p+1} & \cdots\cdots\cdots & c_{p-q+2} \\ \vdots & & \vdots \\ c_{p+q-1} & \cdots\cdots\cdots & c_p \\ \vdots & & \vdots \\ c_{Q-1} & \cdots\cdots\cdots & c_{Q-q} \end{bmatrix} \begin{bmatrix} b_1 \\ b_2 \\ \vdots \\ b_q \end{bmatrix} \tag{14}$$

and if $q>p$, (14) is replaced by

$$\begin{bmatrix} c_{p+1} \\ c_{p+2} \\ \vdots \\ c_q \\ \vdots \\ c_Q \end{bmatrix} = - \begin{bmatrix} c_p & c_{p-1} \cdots\cdots c_1 & -1 & 0 \cdots\cdots 0 \\ c_{p+1} & c_p \cdots\cdots c_1 & -1 & 0 \cdots 0 \\ \vdots & & & \\ c_{q-1} & c_{q-2} \cdots\cdots\cdots & c_1 & -1 \\ \vdots & & & \vdots \\ c_{Q-1} & \cdots\cdots\cdots\cdots & & c_{Q-1-q} \end{bmatrix} \begin{bmatrix} b_1 \\ b_2 \\ \vdots \\ b_q \end{bmatrix} \tag{15}$$

It is noted that the matrices in (14) and (15) are not square, and the pseudo-inverse [9] is employed to compute the b_i from these equations.

When only $z_n = x_n + \xi_n$ is available, the $R_{xx}(\cdot)$ required for identification is estimated from

$$\hat{R}_{zz}(\ell) = \frac{1}{N-\ell} \sum_{n=\ell}^{N-1} z_n \, z_{n-\ell} \triangleq \hat{R}_{xx}(\ell) \tag{16}$$

Now

$$E\{\hat{R}_{zz}(\ell)\} = R_{xx}(\ell) \qquad \text{for } \ell \neq 0 \tag{17}$$

However, if $\ell=0$ in (16), then

$$E\{\hat{R}_{zz}(0)\} = R_o + \sigma_\xi^2 \tag{18}$$

where $R_o \triangleq R_{xx}(0)$ and σ_ξ^2 is the variance of the noise sequence $\{\xi_n\}$. Thus when the output is noise corrupted, (16) will still give unbiased estimate of $R_{xx}(\ell)$ for $\ell \neq 0$ but the estimate for R_o is biased, and using this estimate in (11) will give erroneous results.

A new scheme is now presented for ARMA identification using only $\{z_n\}$. First note from (4) that if $q>p$, both linear dependency and then independance again will have occurred in the ODA before the column containing $R_{xx}(0)$, the a^{th} column, is reached. Additionally, if $p \geq q$, linear dependency will occur before the a^{th} column but linear independence will not occur before the a^{th} column. Hence, without the a^{th} and the following $(a+1)^{th}, \dots, (a+b)^{th}$ columns which all contain R_o, we can still deduce from (4) the following:

(i) the order p, and

(ii) the order q if $p \geq q$, or

(iii) the information that $q>p$.

With p estimated, the a_k coefficients can be obtained, without the need of R_o, by the High Order Yule-Walker Equation (HOYWE) [9]

$$\grave{a} = \Sigma^{-1} g \tag{19}$$

where $\grave{a}^T = [\grave{a}_1 \; \grave{a}_2 \dots \grave{a}_p]$ and the estimate of a_k are

$$\Sigma = \begin{bmatrix} \hat{R}_{xx}(p) & \hat{R}_{xx}(p-1) \cdots\cdots & \hat{R}_{xx}(1) \\ \hat{R}_{xx}(p+1) & \hat{R}_{xx}(p) \cdots\cdots & \\ \vdots & & \\ \hat{R}_{xx}(2p-1) & \cdots\cdots\cdots & \hat{R}_{xx}(p) \end{bmatrix}, \quad g = \begin{bmatrix} \hat{R}_{xx}(p+1) \\ \vdots \\ \vdots \\ \hat{R}_{xx}(2p) \end{bmatrix} \tag{20}$$

The next step is the estimation of R_o. It involves the introduction of a range of trial R_o, starting from the maximum of the $\hat{R}_{zz}(\ell)$, $\ell \neq 0$, denoted $\hat{R}_M$ to $\hat{R}_{zz}(0)$ obtained from (16) with $\ell=0$. The bounds for this range come from the inequalities $R_o \geq R_M$, a property of the autocorrelation function, and $R_{zz}(0) \geq R_o$, as seen from (18). The actual number of R_o in the range depends on the resolution required. In the simulation, R_o is computed by

$$\hat{R}_o^{(j)} = \hat{R}_M + \frac{j(\hat{R}_{zz}(0) - \hat{R}_M)}{K} \tag{21}$$

with $j=0,1,\dots,K$ and $K=20$. For each $\hat{R}_o^{(j)}$, a set of c_ℓ is obtained from (9), by replacing the $R_{xx}(\cdot)$ in (11) by their respective estimates from (16) and $R_{xx}(0)$ by $\hat{R}_o^{(j)}$. Then a set of b_i is computed from either (14) or (15), depending on whether $p \geq q$ (q is not known but set equal to p in (14)) or $q>p$ (then both p and q would have already been determined from (4)). Finally, a set of a_k is obtained from (13). Denote these as $\tilde{a}_k$. These $\tilde{a}_k$ are compared against the $\hat{a}_k$ obtained earlier from (19) by the cost function

$$J = \sum_{k=1}^{p} (\hat{a}_k - \tilde{a}_k)^2 \tag{22}$$

The $\hat{R}_o^{(j)}$ that produces the smallest J is taken as the best estimate $\hat{R}_o^*$. If $q>p$, the identification is complete at this point with (p,q) determined, $\hat{a}_k$ computed from (19) and the $\hat{b}_i$ chosen from the set corresponding to $\hat{R}_o^*$. If $p \geq q$, $\hat{R}_o^*$ is put into the ODA in (4) from which q is determined. Then the b_i are calculated from (14).

To summarize, given only $\{z_n\}$, the identification is as follows:

(i) Form the ODA in (4) with the correlation estimates (16) in place of the true $R_{xx}(\cdot)$.

(ii) Do a ℓ.d. check on the ODA columns by the Gram-Schmidt Orthonormalization (GSO) procedure [4] to get the order p.

(iii) Continue the GSO procedure for linear independency check and if $q>p$, q is also determined before the column containing $\hat{R}_{xx}(0)$ is reached. Then go to (iv). Otherwise $p \geq q$ and q cannot be determined at present. Go to (v).

(iv) Here (p,q) is determined. Compute $\hat{a}_k$ from (19). For each $\hat{R}_o^{(j)}$ in (21), obtain estimates $\tilde{a}_k$ and $\tilde{b}_i$ through (9), (15) and (13) and J from (22). The $R_o^{(j)}$ that gives the smallest J is the best estimate of $R_{xx}(0)$ and the $\tilde{b}_i$ corresponding to that $\hat{R}_o^{(j)}$ are the b_i estimates.

(v) Here it is known that $p \geq q$ and p is determined. Compute $\hat{a}_k$ from (19). For each $\hat{R}_o^j$ in (21), obtain estimates $\tilde{a}_k$ and $\tilde{b}_i$ through (9), (15) and (14). Since q is set to p in (14), the matrix there may not have full rank. This will not pose any problem, however, as the pseudo-inverse solution [9] will provide the best fit. Compute J and choose the $\hat{R}_o^{(j)}$ that gives the smallest J. Put this $\hat{R}_o^{(j)}$ in (4) to determine q. Then recompute the b_i from (14) with the proper q.

III. SIMULATION RESULTS

Two ARMA processes, one has $q>p$ and the other $p>q$, were selected to evaluate the identification scheme. In both examples, the order $Q=20$ was used in (7). The first example is ARMA (1,2) given by

$$x_n = 0.75\, x_{n-1} + W_n - 0.682\, W_{n-1} + 0.578\, W_{n-2}$$

$$z_n = x_n + \xi_n \tag{23}$$

The sequences $\{W_n\}$ and $\{\xi_n\}$ are outputs of independent gaussian random number generators with variances adjusted to give a desired signal to noise ratio, defined in db as

$$\mathrm{SNR} = 10 \log \frac{\sum_{n=0}^{N-1} x_n^2}{\sum_{n=0}^{N-1} \xi_n^2}$$

where N is the number of data points. The results of order determination from $\{z_n\}$ are in Table 1. For such a lower order system, 1000 points are sufficient to achieve reasonable success in order determination. Indeed, although not shown in the table, correct orders were found in all cases for SNR 3.7 db when 5000 points were used.

The other example is ARMA (3,2) given by

$$x_n = 0.1\, x_{n-1} + 0.45\, x_{n-2} - 0.3\, x_{n-3} + W_n + 0.15\, W_{n-1} - 0.3\, W_{n-2}$$

$$z_n = x_n + \xi_n \tag{24}$$

Because of the higher order in this example, more data points are needed to attain the same degree of accuracy in order determination as in the first example. Table 2 lists the results where it is seen that for low SNR, obtaining the proper orders is still very difficult even with 10,000 points.

A plot of the function J of (22) against $\hat{R}_o^{(j)}$ for these two processes, in Fig. 1(a) and (b), reveal a surprising property of this function. For ARMA (1,2), J keeps on decreasing until R_o, the true $R_{xx}(0)$, is reached. After that, J remains relatively constant. In contrast, for the ARMA (3,2) process, J increases again after R_o. Further investigations with other processes confirm the following behavior of J. For $q \geq p$ processes, the general shape of J is as shown in Fig. 1(a) while for $p>q$ processes, J attains a minimum at R_o. This particular property of J, as seen in Fig. 1(a), suggests that if $q \geq p$, using any wrong $\hat{R}_o^{(j)}$, as long as it is greater than R_o, will give good estimates of the $\tilde{a}_k$, remembering that J measures the similarities between $\hat{a}_k$ and $\tilde{a}_k$. This observation can be explained by an examination of the method of [7]. Let $R_{xx}(0) \gg R_{xx}(\ell)$, $\ell \neq 0$, in (9) and (11). Then R in (11) is approximately a diagonal matrix with identical diagonal elements $R_{xx}(0)$ and the c_i from (9) will be approximately equal to $R_{xx}(i)/R_{xx}(0) \ll 1$. Putting these c_i in (14) and (15) and cancelling out $R_{xx}(0)$, it is seen that these equations are the same as the HOYWE of (19), except with a difference in sign, i.e., $b_i \simeq -a_i$. Further, since $|c_i| \ll |b_i|$, from (13), $\tilde{a}_i \simeq -b_i \simeq \hat{a}_i$, resulting in a J function that does not increase after R_o. In contrast, for $p>q$, the coefficients $b_{q+1}, \ldots, b_p$ are set to zero in (13). Thus $\tilde{a}_{q+1}, \tilde{a}_{q+2}, \ldots, \tilde{a}_p$ will not equal $\hat{a}_{q+1}, \ldots, \hat{a}_p$ and J will increase again after R_o, as shown in Fig. 1(b). As mentioned, these conjectures were substantiated by simulation investigations with other processes.

The parameter estimation results are summarized in Tables 3 and 4. In each example, several independent computer runs were conducted and for 25 of those that gave the correct order determination, the coefficient estimates were recorded. Their means and one standard deviations are the entries in the tables.

IV. CONCLUSIONS AND MODIFICATIONS

This paper has presented a scheme for ARMA system identification given only noise corrupted measurements $\{z_n\}$. Because of noise, summing squared samples z_n results in a biased estimate of R_o. By comparing the a_k coefficients obtained from two different methods, one requiring R_o and the other does not, a range of $\hat{R}_o^{(j)}$ was checked to arrive at the proper estimates. Once the major difficulty of obtaining R_o is removed, the ARMA identification problem is solved via the methods of [4] and [7]. Simulation results have verified the procedures in Section II and confirmed the validity of the main idea.

Several variations of this scheme are possible. If $p>q$, an alternate estimate for R_o is available from the equation

$$\hat{R}_{xx}(p) = \sum_{k=1}^{p-1} \hat{a}_k \hat{R}_{xx}(p-k) + \hat{a}_p \hat{R}_{xx}(0) \tag{25}$$

We can also estimate R_o by comparing the coefficients b_i instead of a_k in J. For each $\hat{R}_o^{(j)}$, a set of $\tilde{b}_i$ is computed from (14) or (15) as described previously. A different set, $\hat{b}_i$, can also be computed from (13) using the $\hat{a}_k$ from (19). Now for $J = \Sigma(\hat{b}_i - \tilde{b}_i)^2$, the $\hat{R}_o^{(j)}$ that minimizes J is the best estimate for R_o. Finally from (2), define the residual sequence as

$$r_n = x_n - \sum_{k=1}^{p} a_k x_{n-k} = \sum_{i=0}^{q} b_i W_{n-i} \tag{26}$$

and approximate it by the AR process

$$r_n \simeq \sum_{\ell=1}^{Q} c_\ell r_{n-\ell} + W_n \tag{27}$$

so that (26) becomes

$$r_n \simeq \sum_{i=0}^{q} b_i \left[r_{n-i} - \sum_{\ell=1}^{Q} c_\ell r_{n-i-\ell} \right] \tag{28}$$

Given $\hat{a}_k$, and a range of estimate of σ_ξ^2, $\hat{R}_\xi^{(j)}$, together with $\hat{R}_{xx}(\cdot)$, the autocorrelation functions of r_n, $R_{rr}(\cdot)$, can be calculated from (26). Following the procedure in Section II, starting from (9), a set of $\tilde{a}_k$ is found for each $\hat{R}_\xi^{(j)}$. Comparing them against $\hat{a}_k$ will similarly provide an estimate of σ_ξ^2 and thus R_o. All these variations have been verified by simulation. Their performance is comparable to that of the original scheme although the estimate of $\hat{R}_{xx}(0)$ from (25) is consistently less reliable. This is probably caused by the division operation (by $\hat{a}_p$) needed to obtain $\hat{R}_{xx}(0)$. Errors in $\hat{a}_p$ affect directly $\hat{R}_{xx}(0)$.

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SNR in db	No. of Correct Determination
no noise	14
22.79	14
8.81	11
3.7	8

TABLE 1. ARMA (1,2) Order Determination, 1000 points, 25 runs

SNR in db	No. of Correct Determination	
	5000 Points	10,000 Points
no noise	11	23
22.79	12	23
8.81	7	20
3.7	7	3

TABLE 2. ARMA (3,2) Order Determination, 25 runs each

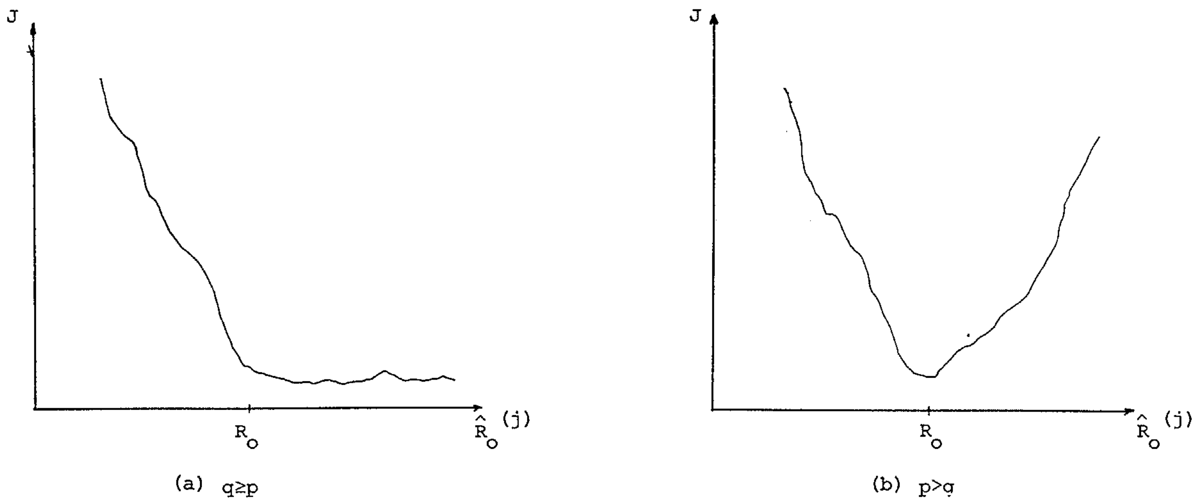

FIGURE 1. Plot of J vs $\hat{R}_o^{(j)}$

No. of Points	SNR	$\hat{a}_1$	$\tilde{b}_1$	$\tilde{b}_2$	Parameter
		0.75	-0.682	0.578	True Value
1,000	no noise	.7584 .0486	-.7042 .0613	.6594 .1201	
	22.79	.7557 .0493	-.6941 .0480	.6241 .0819	
	8.81	.7603 .0545	-.6935 .0635	.6365 .1108	
	3.7	.7760 .0632	-.6527 .1185	.5871 .1563	MEAN Std. Dev.
5,000	no noise	.7461 .0234	-.6957 .0272	.6387 .0713	
	22.79	.7463 .0237	-.6916 .0262	.6171 .0615	
	8.81	.7478 .0250	-.6764 .0299	.5649 .0394	
	3.7	.7512 .0276	-.6636 .0647	.5614 .0909	

TABLE 3. Parameter Estimation ARMA (1,2), 25 runs

No. of Points	SNR	$\hat{a}_1$	$\hat{a}_2$	$\hat{a}_3$	$\tilde{b}_1$	$\tilde{b}_2$	Parameter
		0.1	0.45	-0.3	0.15	-0.3	True Value
5,000	no noise	.1027 .0870	.4456 .0736	-.3235 .0619	.2073 .1098	-.2305 .1086	
	22.79	.1031 .0868	.4466 .0734	-.3236 .0622	.2035 .1157	-.2337 .1126	
	8.81	.0964 .0944	.4441 .0794	-.3392 .0783	.1765 .1333	-.2495 .1348	
	3.7	.0693 .1216	.4218 .1055	-.4011 .1374	.1593 .1247	-.2394 .1564	MEAN Std. Dev.
10,000	no noise	.0950 .0572	.4481 .0550	-.3100 .0281	.2049 .1007	-.2464 .0963	
	22.79	.0936 .0583	.4467 .0549	-.3113 .0283	.2018 .1008	-.2498 .0959	
	8.81	.0854 .0716	.4384 .0640	-.3242 .0393	.1755 .1117	-.2684 .1117	
	3.7	.0685 .1037	.4207 .0932	-.3584 .0754	.1997 .1647	-.2268 .1720	

TABLE 4. Parameter Estimation ARMA (3,2), 25 runs