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NOISE CANCELLATION FOR NARROWBAND INTERFERENCES USING SPARSE ADAPTIVE SYSTEMS.

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RESUME

Dans cet article est developpee une approche, de l'attenuation, dans un signal, des interferences multiples et a bande etroite; cette approche utilise, sous une forme modifiee la methode d'annulation variable du bruit. La methode proposee est basee sur l'idee d'un filtre variable 'clairseme' c'est a dire possedant un nombre de coefficients relativement bas pour rapport a sa longueur ou l'intervalle de temps. L'implementation de ce filtre est identique a elle utilisee pour l'annulation variable du bruit. On suppose que en plus des mesures (primaires) du signal et bruit, on dispose egalement de mesure (reference) possedant pratiquement les meme ensemble d'interferences que les precedentes.

La difference essentielle entre les deux approches reside dans le caractere 'clairseme' du filtre. Les proprietes de ce filtre sont developpees en relation avec le probleme de l'annulation multi tonale et sont critiquees et comparees a celles de l'annulation variable du bruit conventionnelle. La methode d'annulation variable du bruit s'est montre une approche puissante et flexible de l'attenuation des interferences a seul composant tous les montrons qu'en generale, cette methode donne de moins bons resultats pour l'annulation de multiples interferences.

Dans la theorie existante, le systeme est remplace par une fonction de transfert lineaire entre l'entree primaires et la sortie.

Dans le cas d'un ensemble de sinusoides qui interferent avec le signal la fonction de transfert a la forme de M filtres 'notch' en parallele et centres sur les frequences d'interference. Cette description a l'avantage d'etre simple mais n'est en general qu'une approximation, et la precision de l'approximation decroit lorsque le nombre d'interference augmente.

Dans cet article une presentation plus generale de la reponse est attenee, bien que demeurant toujours une approximation.

Cette description est plus complete que la theorie existant (dans laquelle la reponse consiste en un compement lineaire et deux composants non-lineaires), mais elle a l'avantage de fournir une description satisfaisante de la reponse dans le cas de multiples interferences. Une description similaire 'generalisee' de la reponse est egalement developpee pour le filtre variable 'clairseme'. Utilisant a la fois theorie et simulation, on montre que les performances du filtre variable 'clairseme' sont en general superieures a celles de filtre d'annulation du bruit conventionnelle.

SUMMARY

This paper develops an approach to the attenuation of multiple narrowband interferences in a signal, using a modified form of adaptive noise cancellation. The method proposed is based on the idea of a 'sparse' adaptive filter, that is, one with relatively few coefficients in relation to its length or time span. The implementation of this filter is identical to that used in conventional adaptive noise cancellation [1]. That is in addition to the (primary) measurement of signal and noise, a second (reference) measurement consisting almost entirely of a set of interferences similar to those of the primary is assumed to be available. The essential difference between the two approaches is the sparse nature of the filter proposed here. The properties of the sparse adaptive filter are developed in relation to the problem of multi-tone cancellation and are compared and contrasted with those of conventional adaptive noise cancellation. The adaptive noise cancelling method has been found to be a powerful and flexible approach for the attenuation of single sinusoidal interferences. We show that in general it performs far less well for multiple interference cancellation. In the existing theory the system is replaced by a linear transfer function between primary input and output. For the case of a set of M interfering sinusoids the transfer function has the form of M parallel notch filters centred on the interfering frequencies. This description has the attraction of simplicity but is generally only approximate, with the accuracy of the approximation decreasing as the number of interferences increases. In this paper a more general (though still approximate) representation of the response is obtained. This description is more complex than the existing theory (with the response consisting of 1 linear and 2 non-linear components), but has the advantage of giving a satisfactory description of the response for multiple interfering tones. A similar 'generalised' description of the response is also developed for the sparse adaptive filter. It is demonstrated using both theory and simulation that the performance of the sparse adaptive filter is generally superior to that of the conventional noise canceller, both in terms of convergence rate and steady-state attenuation. As a further bonus the lower number of coefficients in the sparse filter leads to a reduced computational burden.



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The application of adaptive noise cancellation [1] to the problem of sinusoidal interferences has been examined by Glover [2] who considers a system as shown in Figure 1. The primary input is assumed to consist of the signal plus a set of M interfering sinusoids with amplitudes B_i , phase angles ϕ_i and frequencies ω_i . A reference measurement is assumed to be available and to consist of a set of sinusoids of similar frequencies to those of the primary input but with amplitudes A_i and phases θ_i . The aim is to filter (using an LMS adaptive filter) the secondary (reference) measurement in such a way as to cancel the sinusoidal components from the primary input, $d(n)$. Glover [2] has shown that the response of this adaptive noise cancelling system can be approximated by a linear time-invariant transfer function relating the primary input $d(n)$, to the error $e(n)$. For the case of a single interfering tone ($M=1$) the transfer function of the system has the form;

$$H(z) = \frac{E(z)}{D(z)} = \frac{1-2z^{-1}\cos\omega_0T+z^{-2}}{\left[1-\frac{\alpha LA_0^2}{2}\right]z^{-2}+\left[\frac{\alpha LA_0^2}{2}-2\right]z^{-1}\cos\omega_0T+1} \quad (1)$$

where L is the number of weights in the adaptive filter, α is the adaptation constant associated with the filter and T is the sample interval for the inputs $x(n)$ and $d(n)$. This transfer function is, however, only exact if L satisfies

$$L = \frac{N\pi}{\omega_0 T} \quad ; \quad N = 1, 2, \dots \quad (2)$$

in other cases the representation is only approximate with the accuracy increasing with L. The system has zeroes at $z = e^{\pm i\omega_0 T}$ and for small adaptation rates the poles lie at approximately $z = (1-\alpha LA^2/4)e^{\pm i\omega_0 T}$. That is, on the same radial lines as the zeroes but $\alpha LA^2/4$ inside the unit circle. The composite system is thus a notch centred on frequency ω_0 . The bandwidth of the notch is controlled by the distance of the poles from the unit circle, and thus by α , L and A_0 . In the time domain it is not unreasonable to estimate the convergence behaviour of the system by considering its response to an input consisting of a pure sinusoid
ie $d(n) = B_0 \cos(\omega_0 nT + \phi_0)$
with this input the response in terms of the

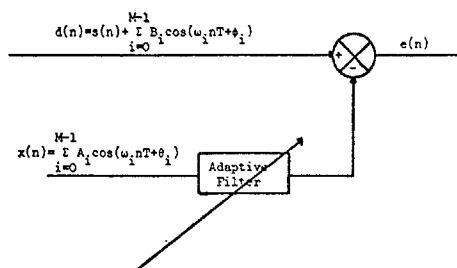


Figure 1: Adaptive noise cancellation for sinusoidal interferences

approximate transfer function can be easily obtained by inverse transforming equation (1), giving

$$e(k) = c \left[1 - \frac{\alpha LA_0^2}{4} \right]^k \cos(\omega_0 kT - \xi) \quad (3)$$

where c and ξ are constants. The approximate transfer function can easily be extended to the case of M input sinusoids (here we

restrict M to 2 to keep the algebra as simple as possible. This is employed throughout the paper, though in all cases the results could easily be generalised).

Consider again Figure 1 with $M=2$, Glover [2] has shown that as for the single sinusoid case, the system can be approximated by a linear time-invariant transfer function of the form:

$$H(z) = \frac{1}{1+G_1(z)+G_2(z)} \quad (4)$$

where

$$G_1(z) = \frac{\alpha LA_0^2}{2} \frac{z^{-1}\cos\omega_0T-z^{-2}}{1-2z^{-1}\cos\omega_0T+z^{-2}}$$

and

$$G_2(z) = \frac{\alpha LA_1^2}{2} \frac{z^{-1}\cos\omega_1T-z^{-2}}{1-2z^{-1}\cos\omega_1T+z^{-2}}$$

The system has zeroes at $z = e^{\pm i\omega_0 T}$ and $z = e^{\pm i\omega_1 T}$ and, neglecting terms involving α^2 poles at

$$z = (1-\alpha LA_0^2/4)e^{\pm i\omega_0 T} \quad \text{and}$$

$$z = (1-\alpha LA_1^2/4)e^{\pm i\omega_1 T}$$

Thus the system corresponds to a pair of notch filters centred at ω_0 and ω_1 . In contrast to the transfer function for a single interfering tone (equation (1)), this system is exact only if it satisfies equation (2) for both ω_0 and ω_1 and for the sum and difference frequencies $\omega_0+\omega_1$ and $\omega_0-\omega_1$. Note that these assumptions will be particularly inappropriate for close frequencies since $\omega_0-\omega_1$ will be small and hence the number of weights required to span π samples will be large. This effect can be illustrated using a simple example. Consider the system of Figure 1 with two interfering tones ($M = 2$), with amplitudes, $A_1, B_1 = 1$, and phases $\theta_1, \phi_1 = 0$ and initially with frequencies $\omega_0 = 750$ Hz and $\omega_1 = 1000$ Hz. The noise canceller was applied with 16 coefficients and an adaptation constant $\alpha = 0.04$. The performance of the canceller is determined by the magnitude of the error signal. The log of this quantity is plotted in Figure 2(a), and as can be seen the log

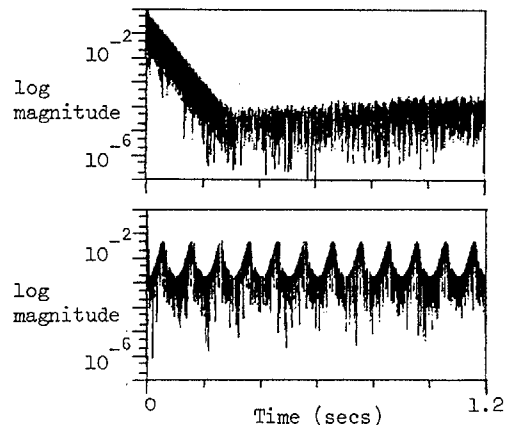


Figure 2: Log magnitude of noise canceller output (error) for $M=2$. a) $\omega_0=750$ Hz, $\omega_1=1000$ Hz, b) $\omega_0=750$ Hz, $\omega_1=740$ Hz

error falls linearly before becoming approximately constant. If the frequencies are now changed to a closely spaced $\omega_0 = 750$ Hz and $\omega_1 = 740$ Hz, say the log error magnitude is changed to that of Figure 2(b). As can be seen, in this case the curve is quite different being periodic and being at a much higher level than the steady-state error in the previous case. Consequently the performance of the

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noise canceller is greatly degraded when the frequencies are closely spaced. This, of course, cannot be predicted from the transfer function theory which does not discriminate between the two cases.

Generalised Description of the Noise Canceller

Response

The extension of the transfer function to M sinusoidal interferences is straightforward [2], and gives a transfer function which consists of M parallel notches at the frequencies ω_i . However, this description is only exact if equation (2) is satisfied for all the frequencies and all the sum and differences thereof. Consequently the transfer function description generally becomes increasingly inaccurate as the number of tones increases, and in fact as we saw in the previous section can be inappropriate with as few as 2 interfering sinusoids. A more general form for the response which is capable of accounting for these extra effects can be obtained. For the single interfering sinusoid this response has the form (the details of the derivation are included in the Appendix):

$$E(z) = H(z)(D(z) + G_1(z))H(ze^{-zi\omega_0 T})D(ze^{-zi\omega_0 T}) + G_2(z)H(ze^{zi\omega_0 T})D(ze^{zi\omega_0 T}) \quad (5)$$

where $H(z)$ is the transfer function of equation (1),

$$G_1(z) = \frac{\alpha A_0^2}{4} \sum_{i=0}^{L-1} e^{i2\phi_i} U(ze^{-i\omega_0 T}) \quad (6)$$

and

$$G_2(z) = \frac{\alpha A_0^2}{4} \sum_{i=0}^{L-1} e^{-i2\phi_i} U(ze^{i\omega_0 T}) \quad (7)$$

$$U(z) = (z-1)^{-1}$$

where $\phi_i = \phi_0 - \omega_0 i T$ and in general $X(ze^a) = z$ transform of $\{x(k)e^{-ak}\}$

This generalised transfer function is depicted in Figure 3. It is clear that the response is no longer a conventional linear transfer function between $d(n)$ and $e(n)$. It is instead the sum of three components. The first component is the usual linear transfer function relation, the other two are obtained by heterodyning $d(n)$ at twice the reference frequency, notch filtering using $H(z)$ rotated in the same manner and then filtering with a first order system. These components are summed and passed through the usual notch. The effect of the non-linear components of the response is, primarily, to create an amplitude scaled and rotated (in frequency) version of the linear response. The magnitude of those components is determined from $G_1(z)$, $G_2(z)$ of equations (6) and (7) by terms of the form:

$$R = \sum_{j=0}^{L-1} e^{i2\phi_j} \quad \text{where } \phi_j = \omega_0 j T$$

so that the magnitude of the non-linear terms is proportional to

$$\left| \sum_{j=0}^{L-1} e^{i2\omega_0 j T} \right| = \left| \frac{\sin L\omega_0 j T}{\sin \omega_0 j T} \right| \quad (8)$$

From this it can be seen that the magnitude of the non-linear components is greatest as $\omega_0 j T \rightarrow 0$ and decreases as $\omega_0 j T \rightarrow \pi/2$ becoming zero in this case,

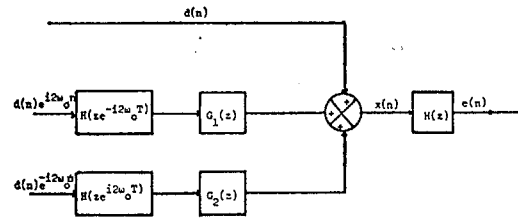


Figure 3: Block diagram for generalised response

so that equation (5) reduces to equation (1). The time domain behaviour of the system can be investigated by assuming that the interference will be the dominant component of the input and neglecting any other input. Substituting for $D(z)$ and $H(z)$ into equation (5), partial fractioning and inverse transforming yields approximately, (see [3]):

$$e(k) = Pr^k \cos(\omega_0 k T + \Theta_1) + kQr^k \cos(\omega_0 k T + \Theta_2)$$

$$\text{where } r = \left(1 - \frac{\alpha I A_0^2}{4}\right) \quad (9)$$

where P , Q , Θ_1 , and Θ_2 are constants. Contrasting this with the comparable equation obtained from the approximate transfer function theory [equation (3)] it is clear that the effect of the second term of equation (9) is always to decrease the convergence rate since $kr^k \rightarrow 0$ more slowly than r^k . Consequently consideration of the linear transfer function description of the response generally leads to over-estimation of the convergence rate.

A similar generalised transfer function can be obtained for multiple interfering tones. For the case of $M = 2$ the response takes the form:

$$E(z) = H(z) \left[D(z) - \sum_{i=1}^8 G_i(z) H_i(z) D_i(z) \right] \quad (10)$$

where

$$G_1(z) = \frac{A_0^2 \alpha}{4} \sum_{i=0}^{L-1} e^{i2\phi_{0i}} U(ze^{-i\omega_0 T}) \quad (11)$$

$$G_2(z) = \frac{A_0^2 \alpha}{4} \sum_{i=0}^{L-1} e^{i2\phi_{0i}} U(ze^{i\omega_0 T}) \quad (12)$$

$$G_3(z) = \frac{A_0^2 \alpha}{4} \sum_{i=0}^{L-1} e^{i2\phi_{1i}} U(ze^{-i\omega_1 T}) \quad (13)$$

$$G_4(z) = \frac{A_0^2 \alpha}{4} \sum_{i=0}^{L-1} e^{i2\phi_{1i}} U(ze^{i\omega_1 T}) \quad (14)$$

$$G_5(z) = \frac{A_0 A_1 \alpha}{4} \sum_{i=0}^{L-1} e^{i(\phi_{0i} - \phi_{1i})} [U(ze^{-i\omega_0 T}) + U(ze^{i\omega_1 T})] \quad (15)$$

$$G_6(z) = \frac{A_0 A_1 \alpha}{4} \sum_{i=0}^{L-1} e^{i(\phi_{1i} - \phi_{0i})} [U(ze^{i\omega_0 T}) + U(ze^{-i\omega_1 T})] \quad (16)$$

$$G_7(z) = \frac{A_0 A_1 \alpha}{4} \sum_{i=0}^{L-1} e^{i(\phi_{1i} + \phi_{0i})} [U(ze^{i\omega_0 T}) + U(ze^{i\omega_1 T})] \quad (17)$$

$$G_8(z) = \frac{A_0 A_1 \alpha}{4} \sum_{i=0}^{L-1} e^{i(\phi_{1i} + \phi_{0i})} [U(ze^{-i\omega_0 T}) + U(ze^{-i\omega_1 T})] \quad (18)$$



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where

$$\begin{aligned}\Phi_{ji} &= \Phi_j - \omega_{ji}T \\ H_1(z)D_1(z) &= H(ze^{-iz\omega_0T})D(ze^{-iz\omega_0T}) \\ H_2(z)D_2(z) &= H(ze^{iz\omega_0T})D(ze^{iz\omega_0T}) \\ H_3(z)D_3(z) &= H(ze^{-iz\omega_1T})D(ze^{-iz\omega_1T}) \\ H_4(z)D_4(z) &= H(ze^{iz\omega_1T})D(ze^{iz\omega_1T}) \\ H_5(z)D_5(z) &= H(ze^{i(\omega_1-\omega_0)T})D(ze^{i(\omega_1-\omega_0)T}) \\ H_6(z)D_6(z) &= H(ze^{-i(\omega_1-\omega_0)T})D(ze^{-i(\omega_1-\omega_0)T}) \\ H_7(z)D_7(z) &= H(ze^{i(\omega_0+\omega_1)T})D(ze^{i(\omega_0+\omega_1)T}) \\ H_8(z)D_8(z) &= H(ze^{-i(\omega_0+\omega_1)T})D(ze^{-i(\omega_0+\omega_1)T})\end{aligned}$$

So that, in addition to containing the linear time-invariant response due to the approximate transfer function $H(z)$, the response also contains components due to the response rotated at both the reference frequencies and components rotated at the sum and difference frequencies. The magnitude of each component is determined by a term of the form:

$$R = \sum_{j=0}^{L-1} e^{+i\psi T} \quad (19)$$

where, in this case, where ψ may be twice either interfering frequency or the sum and difference thereof. This is particularly important in the case of closely spaced interfering frequencies where $\omega_1 - \omega_0 \rightarrow 0$, since referring to equation (8), the magnitude of R will give rise to large non-linear component.

A simple example can be used to demonstrate the validity of the theory developed above. Suppose the noise canceller is employed to cancel two tones ($M = 2$ in Figure 1, with $\omega_0 = 500$ Hz and $\omega_1 = 833$ Hz, say, using 4 coefficients and with the adaptation constant $\alpha = 0.01$. Figure 4(a) shows the canceller output (error) and Figure 5(a) shows its spectrum. From this it can be seen that in

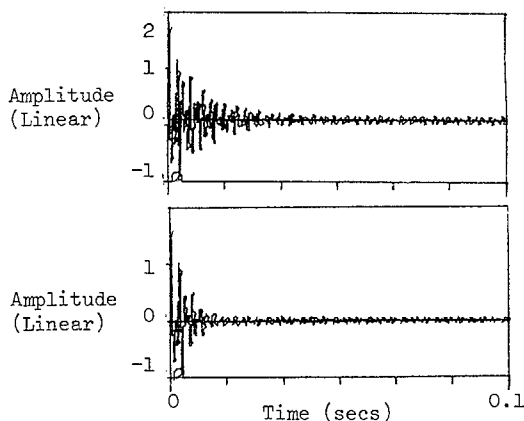


Figure 4: Noise canceller outputs ($M=2, \omega_0=500$ Hz, $\omega_1=833.3$ Hz). a)Conventional ANC, b)Sparse filter

addition to the main peaks due to the partially cancelled sinusoids (during convergence) there are a number of secondary components in the response. Careful inspection of Figure 5(a) shows that these peaks correspond to the non-linear (heterodyned) components predicted by equation (10) occurring as they do, at $\omega_0+2\omega_0$, $-\omega_0+2\omega_1$, $\omega_1+2\omega_0$, etc.

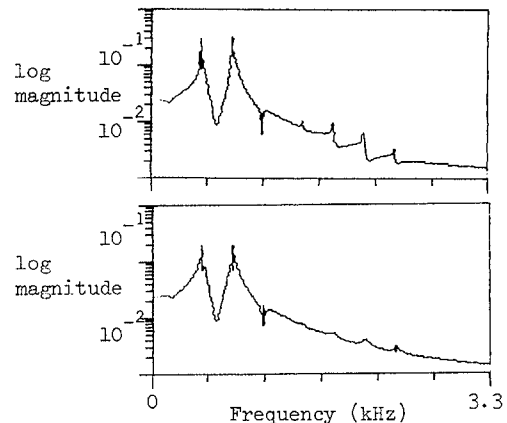


Figure 5: Noise canceller error spectra ($M=2, \omega_0=500$ Hz, $\omega_1=833.3$ Hz. a)Conventional ANC, b)Sparse filter.

Sparse Adaptive Filters

A possible alternative approach to conventional adaptive noise cancellation for narrowband interferences is based on the concept of sparse adaptive filters. A sparse filter is one which has relatively few non-zero coefficients in relation to its length, separated by non-uniform time intervals. For a transversal implementation of such a filter the output would thus have the form:

$$y(n) = f(0)x(n) + f(1)x(n-n_0) + \dots + f(L-1)x(n-n_{L-2})$$

We are concerned with sparse filters whose coefficients are updated using the LMS algorithm. The simplest example of such a filter has just two coefficients (see Figure 6). The idea of two point

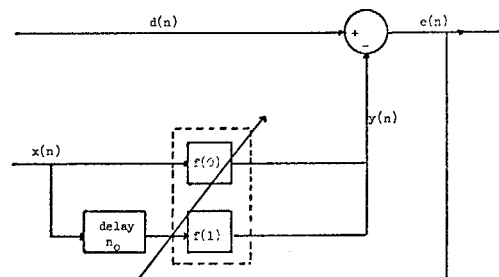


Figure 6: Adaptive noise cancellation using a two point filter

adaptive filters is not new, having been used for some years in processing narrowband signals in antenna arrays. Such filters have also been suggested for notching a single tone in an adaptive noise cancelling system [1]. Here we are proposing the use of a more general $M + 1$ point sparse filter for the cancellation of M tones. The configuration proposed is shown in Figure 7. It is similar to the conventional ANC set up, except for the sparse nature of the input.

The performance of the sparse adaptive filter can be easily evaluated in terms of the approximate linear transfer functions described earlier. For the simplest case (the two point filter) the result is

$$H(z) = \frac{1 - 2z^{-1}\cos\omega_0T + z^{-2}}{(1-\alpha A^2)z^{-2} + (\alpha A^2 - 2)z^{-1}\cos\omega_0T + 1} \quad (21)$$

(which is equivalent to the usual transfer function, with $L = 2$) (see equation (1)). This transfer



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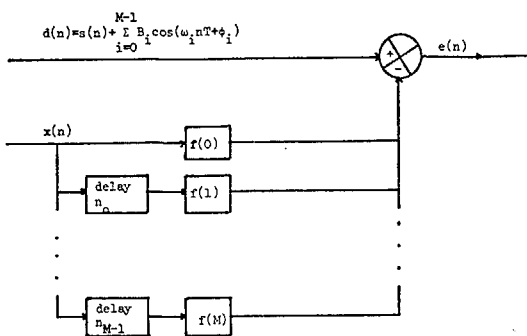


Figure 7: Adaptive noise cancellation using a sparse filter.

function has zeroes at $z = e^{\pm i\omega_0 T}$ and poles approximately at

$$z = (1 - \frac{\alpha A^2}{2}) e^{\pm i\omega_0 T}.$$

Similarly if there are two interfering tones the response has the form:

$$H(z) = \frac{1}{1+G_1(z)+G_2(z)} \quad (22)$$

where

$$G_1(z) = \frac{\alpha A_0^2 (z^{-1} \cos \omega_0 T - z^{-2})}{(1 - 2z^{-1} \cos \omega_0 T + z^{-2})}$$

$$G_2(z) = \frac{\alpha A_1^2 (z^{-1} \cos \omega_1 T - z^{-2})}{(1 - 2z^{-1} \cos \omega_1 T + z^{-2})}$$

However it should be recalled that the transfer function for the multiple sinusoidal interference case will not normally be exact, and that the accuracy of the approximation increases with the number of filter coefficients. Consequently, for the sparse filter the result will have particularly limited accuracy. It is important, therefore, to try to quantify the behaviour of the filter in terms of the more general 'transfer functions' of the previous section. Using the same approach as employed in the Appendix, the generalised response for the 2 point filter is found to satisfy equation (5):

$$E(z) = H(z)D(z) + G_1(z)H(ze^{-2i\omega_0 T})D(ze^{-2i\omega_0 T}) + G_2(z)H(ze^{2i\omega_0 T})D(ze^{2i\omega_0 T}) \quad (5)$$

with

$$G_1(z) = \frac{\alpha A_0^2}{4} (e^{i2n_0 \omega_0 T} + 1) U(ze^{-i\omega_0 T})$$

$$G_2(z) = \frac{\alpha A_0^2}{4} (e^{-i2n_0 \omega_0 T} + 1) U(ze^{i\omega_0 T})$$

That is, the response has exactly the same form as equation (10) for the conventional ANC, only the complex scale factors, $G_1(z)$ for the non-linear terms are different. Similarly for the multi-sinusoidal input case the form of the response is identical but the complex scale factors vary. In this case

$$\sum_{i=0}^{L-1} e^{i2\phi_{0i}} \text{ becomes } (1 + e^{i2n_0 \omega_0 T}) \text{ and}$$

$$\sum_{i=0}^{L-1} e^{i2\phi_{1i}} \text{ becomes } (1 + e^{i2n_1 \omega_0 T}) \text{ and}$$

where it should be recalled that the magnitude of these terms controls both the convergence time of the system, and the magnitude of the spurious components in the frequency response.

Consequently, the analysis of the behaviour of the sparse adaptive filter versus the behaviour of the conventional ANC for multitone cancellation reduces to the comparison of the above terms.

Now, in general

$$\left| \sum_{j=0}^{L-1} e^{i\theta_j T} \right| = \left| \frac{\sin L\theta_j T}{\sin \frac{\theta_j T}{2}} \right|$$

For the sparse form the first four terms are evaluated as $|e^{i\theta}| = 1$, so that the sparse form will have smaller coefficients for frequencies such that $\omega_i < 2\pi/L$. For the sum and difference frequencies similar rules apply, that is, if the sum and/or difference frequencies are less than $2\pi/L$ the sparse formulation will be superior. The difference frequency is particularly relevant if the sinusoids are closely spaced since the difference will then usually be $\ll 2\pi/L$ unless L is very large (in which case the computation may be prohibitive). In addition to the likely superiority of the frequency domain behaviour of the sparse formulation, it will also generally have superior convergence properties. This latter characteristic is due to the fact, asserted earlier, that the convergence is decreased as the filter spacing moves away from π/L at the relevant frequencies. These observations are illustrated using a few simulations. Figure 4 (a) and (b) shows the responses for a single 4-point ANC and a sparse 3-point filter when supplied with 2 sine waves (500 Hz and 833.3 Hz) as both reference and primary. Both filters had the same adaptation constant ($\alpha = 0.01$) but it is clear that the sparse filter converges much more quickly. The modulus frequency responses (for the error) are shown in Figure 5 (a) and (b). It is apparent that the sparse formulation has led to a reduction in the heterodyned components of the response. When compared with the 16-point ANC [Figure 8 (a) and (b)], the sparse formulation is no longer markedly superior but, of course, now has considerable computational advantages.

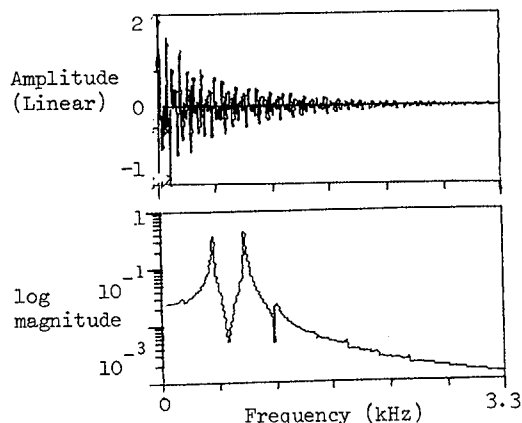


Figure 8: Noise canceller output ($M=2$, $\omega_0=500\text{Hz}$, $\omega_1=833.3\text{Hz}$, 16 weights) a)Time domain b)Modulus spectrum

Conclusions

It has been demonstrated that whilst ANC is a powerful approach to the cancellation of single interfering sinusoids it often is far less successful for multiple interferences. It was found that these differences are not explained by the existing linear transfer function theory, however, the generalised new theory developed in



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this paper gives a more complete description of the response which appears to describe these effects adequately. The sparse adaptive filtering approach proposed as an alternative method to the conventional ANC has been found to be generally superior in terms of both attenuation of the interferences and rate of convergence.

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Appendix

In this appendix we derive the general form for the adaptive noise canceller response of equation (5). Beginning with the LMS update equation

$$f_{k+1}(i) = f_k(i) + \alpha e(k)x(k-i) \quad (A1)$$

where $f_k(i)$ is the k th update, α is the adaptive constant and $x(k)$ is the reference input.

For the single interfering sinusoid

$$\begin{aligned} x(k-i) &= A_0 \cos[\omega_0 kT - \phi_i] \\ &= \frac{A_0}{2} [e^{i\omega_0 kT} e^{-i\phi_i} + e^{-i\omega_0 kT} e^{i\phi_i}] \end{aligned}$$

where $\phi_i = \phi_0 + \omega_0 iT$

Following Glover [2], we may write

$$zF_i(z) = F_i(z) + \frac{\alpha A_0}{2} [e^{i\phi_i} E(ze^{-i\omega_0 T}) + e^{-i\phi_i} E(ze^{i\omega_0 T})]$$

$$F_i(z) = \frac{\alpha A_0}{2} U(z) [e^{i\phi_i} E(ze^{-i\omega_0 T}) + e^{-i\phi_i} E(ze^{i\omega_0 T})] \quad (A2)$$

where $F_i(z)$ is the z -transform of the i th filter coefficient, $U(z) = (z-1)^{-1}$, and where we have used the fact that

$$z \text{ transform of } [e(k)e^{-i\omega_0 k}] = E(ze^{i\omega_0 T})$$

Also

$$y(k) = \sum_{i=0}^{L-1} f_k(i)x(k-i) = \sum_{i=0}^{L-1} y_i \quad \text{say} \quad (A3)$$

where $y_i = f_k(i)x(k-i)$ so that proceeding as before we may obtain $Y_i(z)$ as

$$\begin{aligned} Y_i(z) &= \frac{A_0^2}{4} \alpha [U(ze^{-i\omega_0 T}) + U(ze^{i\omega_0 T})] E(z) \\ &+ \frac{A_0^2}{4} \alpha [e^{i2\phi_i} U(ze^{-i\omega_0 T})] E(ze^{-i2\omega_0 T}) \\ &+ \frac{A_0^2}{4} \alpha [e^{-i2\phi_i} U(ze^{i\omega_0 T})] E(ze^{i2\omega_0 T}) \quad (A4) \end{aligned}$$

Now substituting this equation into $Y(z) = \sum_{i=0}^{L-1} Y_i(z)$

and using $Y(z) = D(z) - E(z)$ we obtain

$$D(z) = R(z)E(z) + G_1(z)E(ze^{-i2\omega_0 T}) + G_2(z)E(ze^{i2\omega_0 T}) \quad (A5)$$

where

$$R(z) = \frac{(1 - \frac{\alpha LA^2}{2})z^{-2} + 2(\frac{\alpha LA^2}{4} - 1)\cos\omega_0 T z^{-1} + 1}{(1 - e^{-i\omega_0 T} z^{-1})(1 - e^{i\omega_0 T} z^{-1})} \quad (A6)$$

$$G_1(z) = \frac{\alpha A^2}{4} \left(\sum_{i=0}^{L-1} e^{i2\phi_i} \right) U(ze^{i\omega_0 T}) \quad (A7)$$

$$G_2(z) = \frac{\alpha A^2}{4} \left(\sum_{i=0}^{L-1} e^{-i2\phi_i} \right) U(ze^{-i\omega_0 T}) \quad (A8)$$

At this point Glover assumes the second and third terms of equation (A5) to be negligible, which leads to the transfer function of equation (1), with $R(z) = H^{-1}(z)$. (Actually, a more straightforward demonstration of this result exists [3].) Here, however, we are concerned to quantify the extra effects.

From (A5) it follows that

$$\begin{aligned} D(ze^{i2\omega_0 T}) &= R(ze^{i2\omega_0 T})E(ze^{i2\omega_0 T}) + G_1(ze^{i2\omega_0 T})E(z) \\ &+ G_2(ze^{i2\omega_0 T})E(ze^{i4\omega_0 T}) \quad (A9) \end{aligned}$$

and similarly for $D(ze^{-i2\omega_0 T})$ so that, from equation (A9)

$$\begin{aligned} \frac{G_2(z)D(ze^{i2\omega_0 T})}{R(ze^{i2\omega_0 T})} &= G_2(z)E(ze^{i2\omega_0 T}) + \\ &+ \frac{G_2(z)G_1(ze^{i\omega_0 T})E(z)}{R(ze^{i2\omega_0 T})} \\ &+ \frac{G_2(z)G_2(ze^{i2\omega_0 T})E(ze^{i4\omega_0 T})}{R(ze^{i2\omega_0 T})} \quad (A10) \end{aligned}$$

and similarly for $D(ze^{-i2\omega_0 T})$

$$\begin{aligned} \frac{G_1(z)D(ze^{-i2\omega_0 T})}{R(ze^{-i2\omega_0 T})} &= G_1(z)E(ze^{-i2\omega_0 T}) + \\ &+ \frac{G_1(z)G_1(ze^{-i\omega_0 T})E(ze^{-i4\omega_0 T})}{R(ze^{-i2\omega_0 T})} \\ &+ \frac{G_1(z)G_2(ze^{i2\omega_0 T})E(z)}{R(ze^{-i2\omega_0 T})} \quad (A11) \end{aligned}$$

Finally, re-arranging these two expressions, substituting into equation (A9) and neglecting $O(\alpha^2)$ yields

$$\begin{aligned} E(z) &= H(z) [D(z) + G_1(z)H(ze^{-2i\omega_0 T})D(ze^{-2i\omega_0 T}) \\ &+ G_2(z)H(ze^{i2\omega_0 T})D(ze^{i2\omega_0 T})] \quad (5) \end{aligned}$$

where, as before $H(z) = 1/R(z)$ is the transfer function.