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VERIFICATION OF A SONAR LOCALISATION BY MEANS OF A MATHEMATICAL MODEL OF AN ANOMALOUS MULTI-MEDIA DETECTION SYSTEM

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RESUME

SUMMARY

The content of the present report, is an approach to verify a conventional sonar localisation of a submerged target by means a two-media integrated detection system (anomalous) which implies the use of sonar localisation as well, but at closer range. The submerged target is considered punctiform in the center of a spherical dominium with radius Ri. A sonar detection system is affected by several error sources distributed along the acoustic path, totally in the water,(Direct Localisation Path, DLP). The integrated Detection System can be conceived in a two-media space where the localisation action can take place, initially in the air, Upper Leg, and then in the water , Down Leg, Fig.1.

The purpose of the study is to evaluate the localisation probability of the above mentioned dominium in function of the "Down Leg" Sonar acquisition radius Ra and of the dominium radius Ri, with respect to the localisation probability of the DLP. The memo shows an approach for the case of Ri = 0 (point dominium). The acquisition or localisation probability Pa , is evaluated as function of Ra, taking in to account the various ipothetical error sources.

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1. GENERALS

The ipothetical system considered can be divided in the main, in three phases: Fig.1.

i) Direct Path Sonar Localisation(here-in after referred as SL1) performed by a sonar system from a main platform (MP) i.e. a hull mounted convenctional sonar on board of a surface ship.

ii) "Upper Leg" (Aerial Phase). As soon as SL1 localisation has been carried out , an additional platform (AP) is rapidly transferred in the vicinity of the detected target , by means, for instance , of a flying - guided carrier. On board of the AP is installed an other sonar system with the task to perform the closer range localisation (SL2).
The AP is dropped in the sea as close as possible to the target position , previously obtained by SL1.

iii) "Down Leg" . As soon as in the water the AP sonar system, start the closer range SL2 operation. Data of SL2 can be sent back to the MP, via radio link , for analysis and comparison with data of SL1.

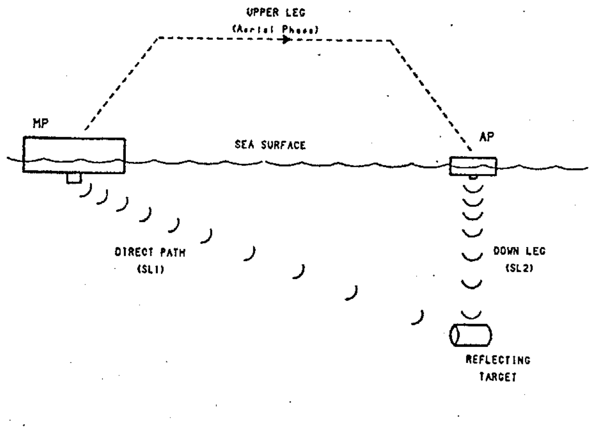

Fig.1 : SKETCH OF THE GENERAL SCENARIO

The AP sonar system is assumed to have some fixed operating ranges (three , expressed in % of the maximum range).

A mathematical model of the whole system has been defined and a programme , simulating its operations , has been implemented in the computer , adopting the Montecarlo Method.

The sequence of events of the system operations, starting with SL1 and ending with SL2, has been repeated N times with N large as much as required from the significance level of the simulation. The N trials of events (or runs) produced are the statistical base to evaluate the operational performance of thesystem.

2. THE SYSTEM SIMULATION MODEL

Fig. 2 shows the block diagram of the system Simulation Model.

2.1 - SL1 Operation.

Target postion is initially determined by SL1.Target data are utilized by the system to compute "Upper Leg" relevant data for the appropriate AP displacement by the flying carrier.
In the simulation model no hypothesis have been made about target strength and doppler effects.

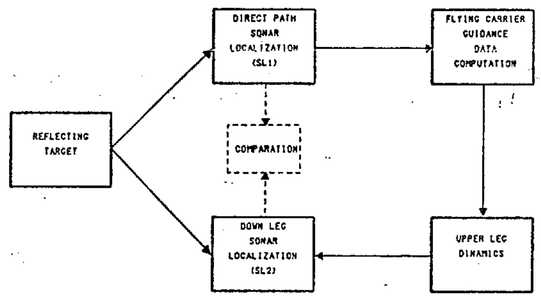

Fig.2 : BLOCK DIAGRAM OF THE SIMULATION MODEL

2.2. - Sonar Accuracy .

The SL1, as the SL2, phase , is affected by various error sources. To put the Model in conditions as close as possible to the actual ones, it is necessary to examinate in details the nature of these error sources.
Assuming the mean duration of an engagement as time reference, it is worth while a distinction between sources of systematic and non-systematic errors (SE and NSE respectively).

- Systematic Errors
  They are originated by erratic movements (roll and pitch) of the sonar and target platforms, around the respective mean positions. Such errors can be considered constant for the duration of the engagement, but variable for longer periods or among different platforms.

- Non - Systematic Errors
  They are mainly originated by the non-point nature of the target and by the relative speed variation between target and sonar platform during sonar emission. As well as the SE , above mentioned, such type of NSE , are generated by instabilities of the detection geometry during the sonar emission. An other source of NSE is due to the acoustic waves dispersion passing through the the medium.

In the general approach to the study, both SE and NSE types have been considered, but in the actual Simulation Model, only the NSE type has been taken in to account.

2.3 - Target Monitoring and Guidance Data Computation.

On the basis of the target data from SL1, the system evaluates the target position and its kinematic characteristics in case of moving target. In this first approach the Simulation Model does not take in to account errors introduced by digital processing of data and errors affecting the various platforms data.

Target data are used as imputs to the algorithm that computes the initial guidance orders for the AP's carrier, provided the feseability of the aerial phase. The Carrier positions , during the flight, are detected by a radar system with known performances , and injected in the guidance algorithm as well.

2.4 - Upper Leg Dynamic.

The Simulation Model takes in to account the dynamic involved in the aerial phase, by properly simulating the Guidance Law, the dynamic peculiarities, and the AP characteristics.

2.4 - SL2 Operation.

The general assumptions described in para 2.1 concen the SL2 as well.
In addition the AP sonar system, performs the SL2 by exploiting prefixed acquisition ranges (Ra) expressed in fractions of the sonar maximum range, as shown in Fig. 3.

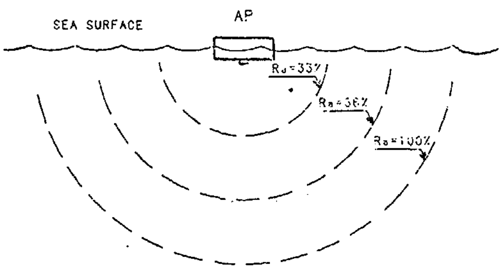

FIG.3 : SKETCH OF PRESETTED Ra VALUES

2.5 - The Reflecting Target Model.

As shown in Fig. 3, the reflecting target model is an underwater point mobile, placed in the center of spherical dominium of radius Ri. Each point of such dominium has identical detection properties.

The Model can be applied to both stationary and moving target.. For the approach described in this report a point-target model (Ri = 0) has been considered.

3. RISULTS

3.1 - Quality Of The Estimate .

As mentioned previously, the simulation study has the final objective to produce an estimate of the target localisation probability by SL2 , provided first localisation data performed by SL1 and in function (as parameter) of the Ra.

In order to ascertain the quality of the estimate, obtained by means the Montecarlo Method, calculations have been carried out to produce diagrams showing relationships among the Accuracy , the Stability of the estimate and number "N" of Montecarlo iterations, taking in to account the assumptions mentioned in Appendix 1.

3.2 - Final Localisation Probability.

Figs. 4, 5, and 6 , show the Localisation Probability performed by the SL2 process, normalized to the SL1 Localisation Probability, for three different predetermined Ra values.

Ra_1 = 33 % of mx value Fig.4

Ra_2 = 66 % of mx value Fig.5

Ra_3 = 100 % of mx value Fig.6

On the abscissa is the ratio between SL1 range and the relative values of Ra .

4. APPENDIX

The Determination of the Number of Iterations for Montecarlo Computation .

In order to obtain , with a given probability, a prescribed accuracy in the estimate of r.m.s. radial error E , the determination of the needed number "N" of iterations is performed as follows:

Assuming S_x^2 and S_y^2 as the estimates of the variances σ_x^2 and σ_y^2 of the coordinates x,y.

We can write:

$$S_x^2 = \frac{1}{N} \sum_{i=1}^{N} (x_i - \bar{x})^2$$

$$S_y^2 = \frac{1}{N} \sum_{i=1}^{N} (y_i - \bar{y})^2 \qquad |1|$$

With errors in x and y normally distributed we have:

$$E = (S_x^2 + S_y^2 + \overline{\Sigma x} + \overline{\Sigma y})^{1/2} \qquad |2|$$

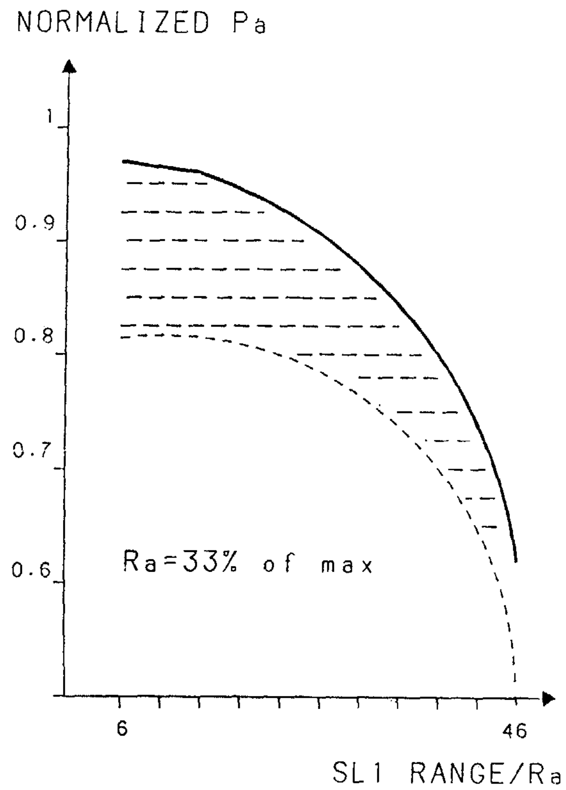

FIG.4

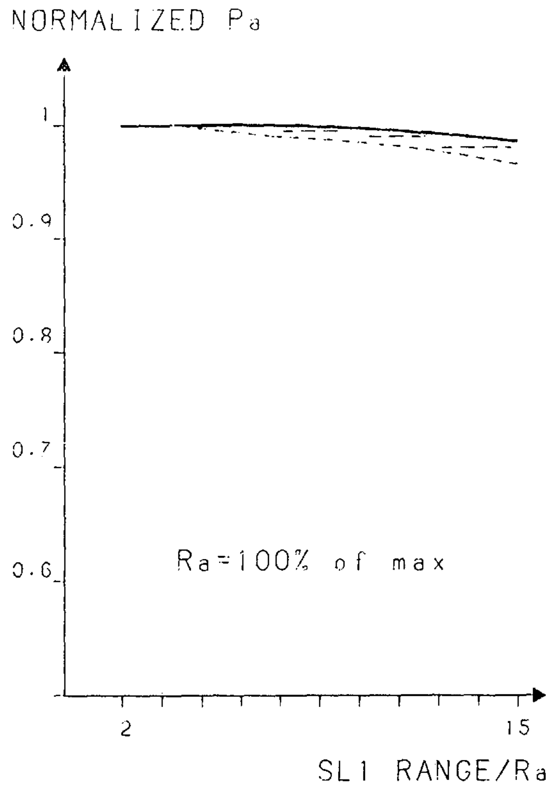

FIG.6

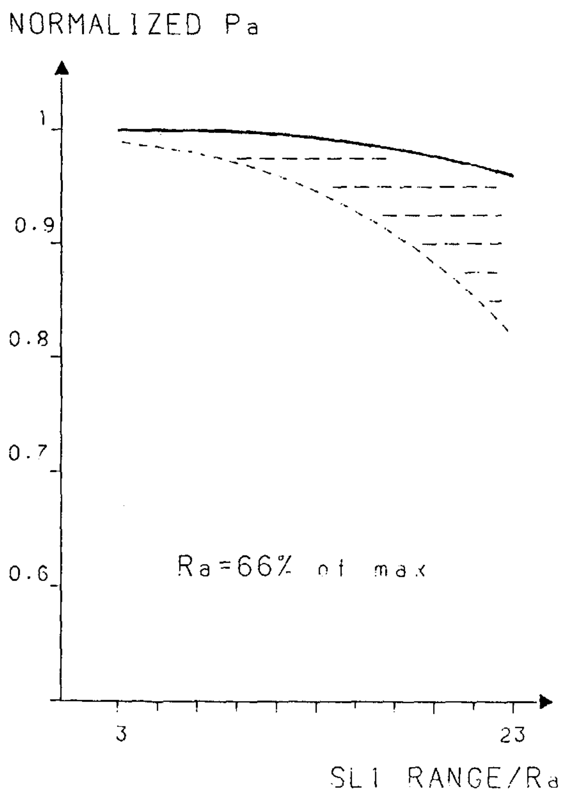

FIG.5

with $\overline{\Delta x}$ and $\overline{\Delta y}$ are reprenting possible systematic errors.

If s^2 is a real estimate of s_x^2 and s_y^2 and σ^2 is the real estimate of σ_x^2 and σ_y^2 , the ratio

$$N\, s^2 / \sigma^2$$

has a χ^2 distribution , with $N - 1$ degrees of freedom. In this case we have that:

the mean value of $N\, s^2/\sigma^2 = N - 1$ |3|

and the variance of $N\, s^2/\sigma^2 = 2(N - 1)$ |4|

For large values of N the χ^2 distribution tends to a normal distribution , Ref. 3.

Consequently the standardized variable:

$$\frac{(N\, s^2/\sigma^2) - (N - 1)}{(2\,(N-1))^{1/2}} \qquad |5|$$

will tend also to a normal distribution, with a mean value = 0 and s.d. = 1.

For large values of N is possible to replace $N-1$ by N, which brings, at the limit, the following expression to be normally distributed with mean-value=0 and s.d. = 1.

For the probability that the relative error S^2 is smaller than e_1 it is possible, eventually write:

$$\text{Prob.}\left\{ \left| S^2/\sigma^2 - 1 \right| < e_1 \right\} =$$

$$D\left\{ e_1 (N/2)^{1/2} \right\} - D\left\{ - e_1 (N/2)^{1/2} \right\} =$$

$$= 2\,D\left\{ e_1 (N/2)^{1/2} \right\} - 1 \qquad |6|$$

In which D denotes the distribution.

The criterion for the estimate is that the number of iteration N used, should give a certain requested probability in % (i.e. 50,80,90,95 % in Figs.7,8) of being within a preset percent (e_1) of the three r.m.s. radial errors. Fig. 9.
Additional assumptions for the estimate are the following:

a. Systematic errors = 0
b. The χ^2 distribution of the ratio NS^2/σ^2 tends to be normal for $N \to \infty$. Ref. 2.

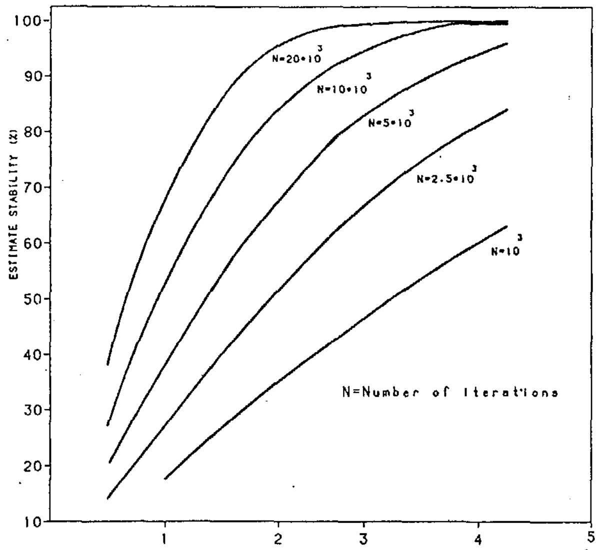

FIG.8 : ISO-ITERATIONS DIAGRAMS

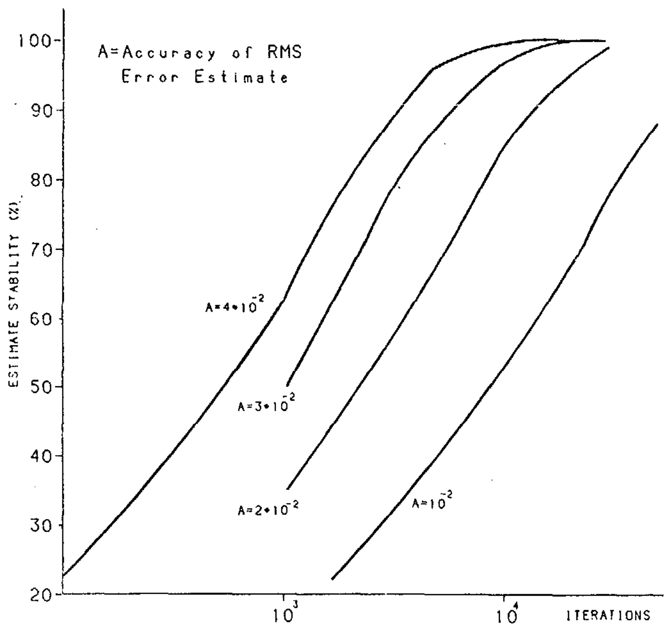

FIG.9 : ISO-ACCURACY DIAGRAMS

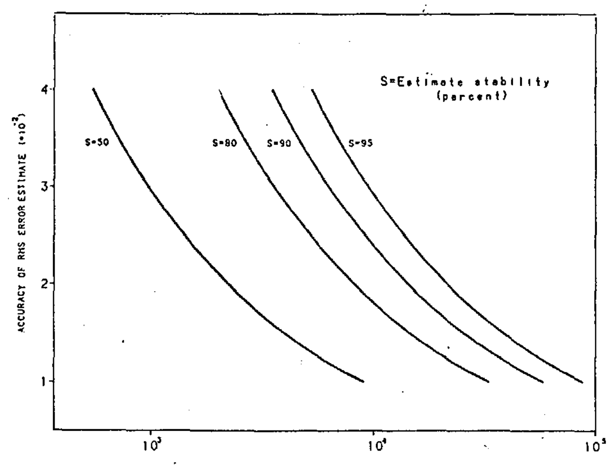

FIG.7 : ISO-STABILITY DIAGRAMS

5. REFERENCES

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