CHAOTIC SIGNAL PROCESSING FOR IDENTIFICATION AND CONTROL

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RÉSUMÉ

Le traitement du signal chaotique consideré dans cet article consiste à la recherche d'orbsites périodiques instables qui existent dans le régime chaotique, à la base d'une série d'échantillons. L'hierarchie d'orbites instables est particulière pour le type d'attracteur et les valeurs de paramètres du système. Charactérisation du comportement chaotique par les orbites instables peut être considéré comme un type d'identification ou de codage du signal. Les orbites instables découvertes dans l'attracteur peuvent être utilisées dans des procédures de contrôle. Les méthodes de stabilisation d'orbites instables dans les systèmes chaotiques sont décrites dans cet article.

ABSTRACT

Chaotic signal processing is understood in this paper as uncovering, from a discrete time series of state variables, of the hierarchy of unstable periodic orbits embedded within the chaotic attractor. Such a characterisation of chaotic behavior can be considered as a particular kind of process identification or signal coding - the hierarchy is unique for a given set of system parameters and signal measured from the system. The uncovered unstable periodic orbits can be used in controlling the chaotic system. We describe the key developments in this area – the methods of stabilisation of any arbitrarily chosen unstable periodic regime existing in the chaotic state.

1. INTRODUCTION

In recent years a varity of tools for processing deterministic and predictable (eg. transient or periodic) or stochastic signals have been developed. None of these methods is fully suitable for processing chaotic signals – a specific class of deterministic signals possessing the property of "sensitive dependence" on initial conditions thus being unpredictable in practical sense [5]. Many real physical systems operate in chaotic regime and signals of this type are often encountered in engineering practice. From the signal processing point of view detection, characterisation, identification, analysis of chaotic signals is interesting also because of many potential applications [10], [12].

In this paper we describe methods developed for processing chaotic signals in view of the problem of controlling chaotic systems. We exploit the fact that unstable periodic orbits are inherently embedded within the strange attractors. Existence of a coutable infinity of such orbits is often recognised as acriterion of existence of chaotic behavior [1], [2], [5], [6]. We analyse unstable periodic orbits characterising chaotic behavior on the basis of chaotic signal (time series) measured from a real systems.

Chaotic signal processing is here understood as uncovering of unstable periodic orbits - we show that this task is feasible and numerically tractable - specific specific algorithms and programs noindent have been developed for this purpose. We describe the details of the control algorithms which further use the computed unstable periodic orbits. We will consider the approach introduced by Ott, Grebogi and Yorke (OGY) [11] – controlling chaos by stabilizing one of the unstable periodic orbits embedded in the chaotic attractor.

2. MODELS OF CHAOTIC SYSTEMS

In our study we assume that the state evolution of the considered chaotic dynamical system is described by either a differential equation $\dot{\boldsymbol{x}}(t) = \boldsymbol{F}_C(\boldsymbol{x}(t))$ or a difference equation $x(n+1) = F_D(x(n))$. Typically a signal we observe is some function of the state variables $\boldsymbol{G}(\boldsymbol{x})$, in many cases is discretized in time and value (sampling, A/D conversion) and can be distorted by noise, fading, interference etc. In many cases we have just one measurable variable and we encounter the problem of reconstructing the state variables (embedding problem). There are interesting issues in this area – the most commonly used is the method of delay coordinates [5].

How can we recognise that a signal measured from some process is chaotic and how can one characterise such a signal. First approach is to calculate so-called metric invariants: Lyapunov exponents, dimensions, entropy, spectrum of singularities, invariant measures etc. The other approach deals with topological properties of trajectories.

3. CHARACTERISING CHAOS BY UNSTABLE PERIODIC ORBITS

Characterisation in terms of unstable periodic orbits belongs to the class of topological methods. The knowledge of the structure of unstable periodic orbits enables us to find an approximation to the curvatures of the nonlinear multidimensional Poincaré map using a continuous polygonal surface made of hyperplanes in such a way that these hyperplanes are tangent to the graph of the map at the unstable periodic points and their slopes are determined by eigenvalues of the Jacobian matrices calculated at these points. One can obtain any needed accuracy of approximation as there exists a countable infinite number of unstable periodic orbits with growing periods and these orbits are dense on the asymptotic strange set [5], [7] — recovering more and more unstable cycles we obtain better approximations.

The main features of the characterisation in terms of unstable periodic orbits are [1], [2], [7], [9]:
- Periodic orbits and their eigenvalues are topologically invariant — different representations of the same system (up to a smooth transformation of coordinates) must preserve their topological properties (a fixed point must remain a fixed point in any representation and the same applies to periodic orbits),
- Periodic orbits constitute a "skeleton" for the attractor — they determine its spatial layout,
- The eigenvalues of closed orbits are metric invariants — they describe the scaling between different pieces of the attractor.
- There exists a hierarchical ordering of unstable periodic orbits — short cycles give good approximations of the strange set.
- Periodic orbits are robust — they vary slowly with smooth parameter variations. The same applies to their eigenvalues.
- Unstable periodic orbits can be successfully extracted from experimental data - specific computational methods have been developed for this purpose and implemented in computer prrograms.

4. UNCOVERING UNSTABLE PERIODIC ORBITS

The numerical procedure for processing of the chaotic signal assumes that we have a series of successive points $\{x_i\}$, $i = 0, 1, ...N$ on the system trajectory measured using some data acquisition procepoint x_n in this time series. We follow the succesive points x_{n+1}, x_{n+2} etc. until we find the smallest k such that $||x_{n+k} - x_n|| < \varepsilon$. It is further claimed that the orbit detected in this manner lies close to the unstable periodic orbit whose period is approximated by that of the detected sequence. (see Auerbach *et all.* [1], Lathrop and Kostelich [7], Cvitanović [2]). Choosing suitable ε a variety of periodic orbits can be uncovered from the experimental signal.

Such an approach has several drawbacks. Firstly, the results strongly depend on the choice of ε and the length of the measured time series. Further, they depend on the choice of norm and number of state variables available. In the experiments we normalised the size of the attractor and used the Euclidean norm. Secondly, the stopping criterion ($||x_{m+k} - x_m|| < \varepsilon$) in the case of discretely sampled continuous-time systems might give wrong results. One can never tell whether all orbits of a given period have been recovered.

In many applications , however, it is sufficient to find only some of the unstable periodic orbits embedded in the attractor — this is the case for example in the Ott, Grebogi and Yorke approach to controlling chaos [6], [11].

Among unstable periodic orbits calculated using the described procedure there are always groups of nearly identical ones. It is very important to introduce a criterion for distinguishing different periodic orbits. This can be done on the basis of calculating the distance between the orbits. The distance between orbits Γ_1 and Γ_2 is defined as: $d_{orb} = \max_{x_i \in \Gamma_1}[\min_{x_k \in \Gamma_2} ||x_k - x_i||]$ Two orbits whose distance is smaller then the prescribed threshold are considered equal. With greater ε more orbits with given period were detected most of them were later recognised as identical - there was no significant difference in the number and shape of different unstable periodic orbits found.

5. DETERMINATION OF ORBIT EIGENVALUES

This procedure follows the guidelines given by Lathrop and Kostelich [7], and also described by Ott *et all.* [11]. Let us take a point x_m belonging to the chosen orbit, laying on a chosen section (Poincaré) plane and a 3ε neigborhood around it. Let $\{x_{3\varepsilon}\}$ be the set of points in this neighborhood. We assume that the dynamics in this small neighborhood is nearly linear and can be aproximated by $x_{i+1} = Ax_i + B\alpha$. A and B can be found using eg. least squares procedure. Matrix A gives an approximation of the Jacobian matrix of the periodic orbit. For the evaluation of B repetition of the whole procedure for slightly changed α is needed. The eigenvalues of the Jacobian matrix can be further evaluated using standard methods.

6. STABILIZATION OF UNSTABLE PERIODIC ORBITS

We restrict the description to three-dimensional continuous dynamical systems. Let us assume that the equations of the system can be written in the form:

$$\frac{d\boldsymbol{x}}{dt} = \boldsymbol{F}(\boldsymbol{x}, p) \tag{1}$$

where p is some accessible system parameter which can be perturbed around its nominal value p_0 with maximal possible perturbation δp_{max}. In the first step we must choose a hyperbolic-type periodic orbit we want to stabilize. It could be one of the unstable orbits foud using the procedure described in the previous section. Once the goal of the control is fixed a Poincaré section for the trajectories of our three dimensional continuous system must be chosen in such a way that it intersects transversally the orbit we want to stabilize. Let us assume that the Poincaré map of the system has the form:

$$\xi_{i+1} = f(\xi_i, p) \tag{2}$$

where ξ_i is the i^{th} piercing of the surface of section by the trajectory. Let us denote the time of the i^{th} intersection by t_i. First we describe the control method for a period-one orbit, which corresponds to a fixed point in the Poincaré map.

Ott-Grebogi-Yorke method

Let us recall briefly the OGY [11] control idea. Let $\xi_F = f(\xi_F, p_0)$ denote the unstable fixed point on the Poincaré section which one wants to stabilize. We use the linear approximation of the Poincaré map near the fixed point ξ_F and the nominal value of control parameter p_0.

$$\delta\xi_{i+1} = \boldsymbol{M}\delta\xi_i + \boldsymbol{w}\delta p_i \tag{3}$$

where $\delta\xi_i := \xi_i - \xi_F$, $\delta p_i := p_i - p_0$, $\boldsymbol{M} := D_\xi f(\xi_F, p_0)$ and $\boldsymbol{w} = \frac{\partial f}{\partial p}(\xi_F, p_0)$. Matrix $\boldsymbol{M}$ is the Jacobian matrix of the periodic orbit. Let e_u, e_s be the unstable and stable eigenvectors of matrix $\boldsymbol{M}$, and λ_u, λ_s the corresponding eigenvalues. Since by assumption ξ_F is hyperbolic then $|\lambda_u| > 1$ and $|\lambda_s| < 1$. Let f_u, f_s be the contravariant basis vectors defined by: $f_s \cdot e_u = f_u \cdot e_s = 0$ and $f_s \cdot e_s = f_u \cdot e_u = 1$. In order to control the system we observe the trajectory until it comes close to the chosen periodic orbit and then modify the control parameter in such a way as to push the trajectory onto the local stable manifold of the periodic point. This condition can be formulated as $f_u \cdot \delta\xi_{i+1} = 0$. From that we can derive the formula for δp_i:

$$\delta p_i = -\frac{\lambda_u}{f_u \cdot \boldsymbol{w}} f_u \cdot \delta\xi_i \tag{4}$$

One has to assume that the generic condition $f_u \cdot \boldsymbol{w} \neq 0$ is satisfied. The control signal $p_i = p_0 + \delta p_i$ is applied in the system only at the very moments when the actual chaotic trajectory passes near the chosen periodic point. Since by assumption the system is ergodic the trajectory will pass arbitrarily close to the chosen point.

Dressler-Nitsche modification

Dressler and Nitsche [4] noticed that when using time-delay coordinates the experimental surface of section map f depends not only on the actual value p_i but also on the preceding value p_{i-1}, i.e.

$$\xi_{i+1} = f(\xi_i, p_{i-1}, p_i) \tag{5}$$

The linearization of the system can be written in the form:

$$\delta\xi_{i+1} = \boldsymbol{M}\delta\xi_i + \boldsymbol{v}\delta p_{i-1} + \boldsymbol{u}\delta p_i \tag{6}$$

where $\boldsymbol{M} = D_\xi f(\xi_F, p_0, p_0)$, $\boldsymbol{v} = \frac{\partial f}{\partial p_{i-1}}(\xi_F, p_0, p_0)$ and $\boldsymbol{u} = \frac{\partial f}{\partial p_i}(\xi_F, p_0, p_0)$. Demanding $f_u \cdot \delta\xi_{i+1} = 0$ one can obtain the condition for δp_i:

$$\delta p_i = -\frac{\lambda_u}{f_u \cdot \boldsymbol{u}} f_u \cdot \delta\xi_i - \frac{f_u \cdot \boldsymbol{v}}{f_u \cdot \boldsymbol{u}} \delta p_{i-1} \tag{7}$$

This control formula could be unstable. In the case $|\frac{f_u \cdot \boldsymbol{v}}{f_u \cdot \boldsymbol{u}}| \geq 1$ the control signal δp_i could grow until it exceeds the maximum allowed perturbance δp_{max}. To avoid this Dressler and Nitsche proposed to find a control law for δp_i such that δp_{i+1} will automatically become zero. This is done by demanding that the system stabilizes in two steps and δp_{i+1} equal zero, i.e. $f_u \cdot \delta\xi_{i+2} = 0$, $\delta p_{i+1} = 0$. One can obtain then the following control law:

$$\delta p_i = -\frac{\lambda_u^2}{\lambda_u f_u \cdot \boldsymbol{u} + f_u \cdot \boldsymbol{v}} f_u \cdot \delta\xi_i - \frac{\lambda_u f_u \cdot \boldsymbol{v}}{\lambda_u f_u \cdot \boldsymbol{u} + f_u \cdot \boldsymbol{v}} \delta p_{i-1} \tag{8}$$

The above control formula has better stability properties than formula (7).

Control of higher-period orbits

There are several possibilities for stabilizing higher-period orbits. The simplest one is to consider an period-m orbit on the Poincaré section as a fixed point of map f^m. All parameters necessary for the control can be obtained in the same way as for period-one orbit. The only difference is that the control signal is applied every m^{th} piercing of the section plane. The main advantage of this approach is that we need to calculate and store in memory very few values necessary for the control. We have found that this method could work properly for low period orbits (2 or 3). For higher-period orbits after m piercings of Poincaré section trajectory could wander away from the desired periodic point and the goal of the control could be lost.

Ott, Grebogi and Yorke [11] proposed a method which overcomes this problem. Let the period-m orbit in the Poincaré section be: $\xi_{F_1}, \xi_{F_2}, \ldots, \xi_{F_m}, \xi_{F_{m+1}} = \xi_{F_1}$. Let

$$\delta\xi_{n+1} = \boldsymbol{M}_n\delta\xi_n + \boldsymbol{w}_n\delta p_n \tag{9}$$

be the linear approximation of the Poincaré map near the point ξ_{F_n} and the nominal value of control parameter p_0.

Let f_u, f_s be the stable and unstable contravariant basis vectors of $\boldsymbol{M}_n$.

Requireing $f_{u,n+1} \cdot \delta\xi_{n+1} = 0$ one can obtain the following control law:

$$\delta p_n = -\frac{f_{u,n+1} \cdot \boldsymbol{M}_n \cdot \delta\xi_n}{f_{u,n+1} \cdot \boldsymbol{w}_n} \tag{10}$$

There is a number of parameters that must be calculated and stored – their number increases linearily with m, the accuracy is independent on the period.

We have performed several experiments of controlling chaos in Chua's circuit. In our experiments we used the implementations of the nonlinear resistor N_R (using two op-amps) proposed by Kennedy and modified the circuit by adding a linear, voltage-controlled resistor in parallel with the nonlinear one. We used a high-performance data acquisition card (Advantech PCL818) to monitor all state variables. Using the recorded time series we were able to find unstable periodic orbits embedded within the chaotic attractor [8, 9]. Calculation of eigenvalues and eigenvectors of periodic orbits as well as determination of control signals was carried out using the developed software package [3]. The control signal was applied to the circuit from the computer using the analog output channel of data acquisation card. We implemented all the described methods in computer programs. A number of experiments in stabilizing periodic orbits embedded within the attractor obtained from real Chua's circuit were performed [3]. The main problem in real implementation is the noise introduced by quantization during A/D conversion of signals. This causes that periodic orbits found and control values calculated and supplied to the circuit are not accurate.

7. CONCLUSIONS

Using the developed application–specific software package we were able to uncover, from discrete time series of state variables (measured from a real physical electronic system or numerical exeriment), the hierarchy of unstable periodic orbits embedded within typical chaotic attractors. This hierarchy (ie. the lengths and the number of orbits of distinct types) is specific to the particular sets of system parameters and gives a characterisation of the attractor which exists for this choice of parameters. Uncovering of the hierarchy of unstable periodic orbits characterising chaotic behavior can be considered as a particular kind of process identification or signal coding - the hierarhy is unique for a given set of system parameters and signal measured from the system.

The detected orbits were further used in the control procedures. In a number of numerical and laboratory investigations [3]. The methods and algorithms described in this paper can be used also in studies of different physical systems.

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