MIMO LINEAR SYSTEM INPUT/OUTPUT RELATIONS FOR CYCLIC HIGHER-ORDER STATISTICS

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RÉSUMÉ

Les relations entrée/sortie entre les statistiques cycliques d'ordre supérieur à deux sont dérivées pour le cas d'un système linéaire variant en temps presque périodiquement et à plusieures entrées et sorties, lorsque les signaux aux entrées sont cyclostationnaires. Deux exemples d'application sont présentés: l'identification d'un système linéaire invariant en temps par des données bruitées et l'évaluation de statistiques cycliques d'ordre supérieur à deux pour quelques signaux couramment rencontrés en communication.

ABSTRACT

Input/output relations in terms of cyclic higher-order statistics are derived with reference to multiinput multioutput linear almost-periodically time-variant systems that are excited by cyclostationary input signals. As examples of application,the problem of linear time-invariant system identification by noisy measurements is considered and the evaluation of higher-order cyclic cross-spectra for signals of interest in communications is carried out.

1. Introduction

In the last two decades, the theory of second-order cyclostationarity (SOCS) of time-series has been developed [1]. For such kind of signals, there exists a quadratic time-invariant transformation that converts into spectral lines the hidden periodicities due to modulation, sampling, and multiplexing operations. The properties of SOCS have played an important role in the development of new signal processing techniques for such purposes as detection, parameter estimation, and waveform estraction, especially in severe noise and interference environments [2-6].

There is a class of signals, however, whose degree of SOCS is low or zero, whereas the degree of higher-order cyclostationarity (HOCS) is substantial, that is, spectral lines with significant power are generated by an Nth-order homogeneous nonlinear time-invariant transformation with $N > 2$. This class includes pulse-amplitude-modulated (PAM) signals with less than 50% excess bandwidth, M-ary phase-shift-keyed (PSK) signals with $M \geq 4$, and severely bandlimited digital signals. For such signals, the theory of HOCS, which has been very recently introduced [7-10], provides a potentially useful tool for signal processing applications.

In the present paper, after a brief introduction (Section 2) on the joint HOCS, the input/output relations for a linear almost-periodically time-variant (LAPTV) multiinput multioutput (MIMO) system excited by time-series exhibiting joint HOCS are derived. The relations are expressed in terms of cyclic temporal cross-moment functions, cyclic spectral cross-moment functions, cyclic temporal cross-cumulant functions, and cyclic cross-polyspectra. The special case of MIMO linear time-invariant (LTI) systems is, then, considered. Moreover, for a single-input single-output (SISO) LTI system the generalization of the cyclic Wiener system identification formula [1] from second-order to higher-order cyclic spectra is stated. Finally, in Section 4 two examples of application of the relations stated in Section 3 are presented.

The analysis framework used throughout the paper is that of the fraction-of-time probability for time-series exhibiting cyclostationarity [11], which obviates the concept of cyclo-ergodicity and, then, avoids some difficulties related to the estimation of the HOCS parameters [9].

2. Joint higher-order cyclostationarity

Let us consider the column vector $\mathbf{x}(t) \triangleq [x_1(t), ..., x_N(t)]^{\mathrm{T}}$ whose components are N not necessarily distinct complex-valued time-series. The N time-series exhibit joint cyclostationarity of order N with cycle frequency $\alpha \neq 0$ if at least one of the Nth-order cyclic temporal cross-moment functions (CTCMF's)

$$R_{\mathbf{x}}^{\alpha}(\boldsymbol{\tau})_N \triangleq \langle L_{\mathbf{x}}(t,\boldsymbol{\tau})_N e^{-j2\pi\alpha t}\rangle \triangleq \langle \prod_{i=1}^{N} x_i^{(*)_i}(t+\tau_i) e^{-j2\pi\alpha t}\rangle \tag{1}$$

is not identically zero. In (1), $\boldsymbol{\tau} \triangleq [\tau_1, ..., \tau_N]^{\mathrm{T}}$, $\langle\cdot\rangle$ denotes infinite time averaging, and $(*)_i$ represents optional conjugation of the ith factor $x_i(\cdot)$ of the Nth-order lag product waveform $L_{\mathbf{x}}(t,\boldsymbol{\tau})_N$.

The magnitude and phase of the function $R_{\mathbf{x}}^{\alpha}(\boldsymbol{\tau})_N$ are the amplitude and phase of the sine-wave component with frequency α contained in the lag product. In the fraction-of-time probability context for time-series that exhibit cyclostationarity,

$$\langle L_{\mathbf{x}}(t,\boldsymbol{\tau})_N\rangle^{\{\alpha\}} \triangleq \sum_{\alpha} \langle L_{\mathbf{x}}(u,\boldsymbol{\tau})_N e^{-j2\pi\alpha u}\rangle e^{j2\pi\alpha t}$$

$$= \sum_{\alpha} R_{\mathbf{x}}^{\alpha}(\boldsymbol{\tau})_N e^{j2\pi\alpha t} \tag{2}$$

is called the Nth-oder temporal cross-moment function (TCMF), and will be denoted by $R_{\mathbf{x}}(t, \boldsymbol{\tau})_N$. In (2), the sums range over all Nth-order cycle frequencies α, and $\langle\cdot\rangle^{\{\alpha\}}$ stands for the almost-periodic-component extraction operator [11].

The N-dimensional Fourier transform of the CTCMF, which is called the Nth-order cyclic spectral cross-moment function (CSCMF), can be written as [8]

$$S_{\mathbf{x}}^{\alpha}(\boldsymbol{f})_N = \overline{S}_{\mathbf{x}}^{\alpha}(\boldsymbol{f}')_N \delta(\boldsymbol{f}^{\mathrm{T}}\mathbf{1} - \alpha), \tag{3}$$

where $\boldsymbol{f} \triangleq [f_1, ..., f_N]^{\mathrm{T}}$, $\mathbf{1}$ is the vector $[1, ..., 1]^{\mathrm{T}}$, $\delta(\cdot)$ is the Dirac delta function, and prime denotes the operator that transforms the vector $\boldsymbol{u} \triangleq [u_1, ..., u_K]^{\mathrm{T}}$ into $\boldsymbol{u}' \triangleq [u_1, ..., u_{K-1}]^{\mathrm{T}}$. The function $\overline{S}_{\mathbf{x}}^{\alpha}(\boldsymbol{f}')_N$, referred to as the reduced-dimension CSCMF (RD-CSCMF), can be expressed as the $(N-1)$-dimensional Fourier transform of

$$\overline{R}_{\mathbf{x}}^{\alpha}(\boldsymbol{\tau}')_N \triangleq R_{\mathbf{x}}^{\alpha}(\boldsymbol{\tau})_N \,|_{\tau_N=0}, \tag{4}$$

which is the reduced-dimension CTCMF (RD-CTCMF).

Let us note that both $S_{\mathbf{x}}^{\alpha}(\boldsymbol{f})_N$ and $\overline{S}_{\mathbf{x}}^{\alpha}(\boldsymbol{f}')_N$ in general contain products of impulses and, then, are not well-behaved functions. This stems from the fact that the function $\overline{R}_{\mathbf{x}}^{\alpha}(\boldsymbol{\tau}')_N$ is not in general absolutely integrable [9]. However, a well-behaved function in the spectral-frequency domain can be introduced starting from the Nth-order temporal cross-cumulant function (TCCF)

$$C_{\mathbf{x}}(t, \boldsymbol{\tau})_N$$

$$\triangleq (-j)^N \frac{\partial^N}{\partial\omega_1 \cdots \partial\omega_N} \log_e \langle \exp\{j \sum_{k=1}^{N} \omega_k x_k(t+\tau_k)\}\rangle^{\{\beta\}} \,|_{\boldsymbol{\omega}=\mathbf{0}}$$

$$= \sum_{\mathsf{P}} \left[(-1)^{p-1}(p-1)! \prod_{i=1}^{p} R_{\mathbf{x}_{\mu_i}}(t, \boldsymbol{\tau}_{\mu_i})_{|\mu_i|} \right], \tag{5}$$

where $\boldsymbol{\omega} \triangleq [\omega_1, ..., \omega_N]^{\mathrm{T}}$, P is the set of distinct partitions of $\{1, ..., N\}$, each constituted by the subsets $\{\mu_i : i = 1, ..., p\}$, $|\mu_i|$ is the number of elements in μ_i, and $\mathbf{x}_{\mu_i}$ is the $|\mu_i|$-dimensional vector whose components are those of $\mathbf{x}$ having indices in μ_i. In fact, taking the N-dimensional Fourier transform of the coefficient of the Fourier series expansion of the almost-periodic function (5)

$$C_{\mathbf{x}}^{\beta}(\boldsymbol{\tau})_N \triangleq \langle C_{\mathbf{x}}(t, \boldsymbol{\tau})_N e^{-j2\pi\beta t}\rangle, \tag{6}$$

which is referred to as the Nth-order cyclic temporal cross-cumulant function (CTCCF), one obtains the Nth-order cyclic spectral cross-cumulant function (CSCCF) $P_{\mathbf{x}}^{\beta}(\boldsymbol{f})_N$. It can be written as [8]

$$P_{\mathbf{x}}^{\beta}(\boldsymbol{f})_N = \overline{P}_{\mathbf{x}}^{\beta}(\boldsymbol{f}')_N \delta(\boldsymbol{f}^{\mathrm{T}}\mathbf{1} - \beta), \tag{7}$$

where the Nth-order cyclic cross-polyspectrum (CCP) $\overline{P}_{\mathbf{x}}^{\beta}(\boldsymbol{f}')_N$ is the $(N-1)$-dimensional Fourier transform of

$$\overline{C}_{\mathbf{x}}^{\beta}(\boldsymbol{\tau}')_N \triangleq C_{\mathbf{x}}^{\beta}(\boldsymbol{\tau})_N \,|_{\tau_N=0}, \tag{8}$$

which is the reduced-dimension CTCCF (RD-CTCCF). The cyclic cross-polyspectrum is a well-behaved function under the mild conditions that the time-series $x_i(t)$ $(i = 1, ..., N)$ are asymptotically independent (so that $\overline{C}_{\mathbf{x}}^{\beta}(\boldsymbol{\tau}')_N \to 0$ as $|\boldsymbol{\tau}'| \to \infty$) and, moreover, there exists an $\epsilon > 0$ such that $|\overline{C}_{\mathbf{x}}^{\beta}(\boldsymbol{\tau}')_N| = o(|\boldsymbol{\tau}'|^{-N+1-\epsilon})$ as $|\boldsymbol{\tau}'| \to \infty$.

Finally, we note that in Section 3, for analytical simplicity, the input/output relations for LAPTV systems are first derived in terms of cyclic moments and, then, in terms of cyclic cumulants, by exploiting the relationship between moments and cumulants.

3. MIMO LAPTV input/output relations

Let us consider an N-input M-output LAPTV system and express the elements of its impulse response matrix by the Fourier series expansion

$$h_{mn}(t+\tau_m, t) = \sum_{\nu_{mn}} h_{\nu_{mn}}(\tau_m) e^{j2\pi\nu_{mn}t}, \tag{9}$$

where $m = 1, ..., M$, $n = 1, ..., N$, and the sum is over all harmonics of the almost-periodic function $h_{mn}(t+\tau_m, t)$.

The CTCMF of order M at the cycle frequency α of the M outputs $y_m(t)$ can be obtained by (1) and (9):

$$R_{\mathbf{y}}^{\alpha}(\boldsymbol{\tau})_M \triangleq \langle \prod_{m=1}^{M} y_m(t+\tau_m) e^{-j2\pi\alpha t}\rangle$$

$$= \sum_{i_1,...,i_M=1}^{N} \sum_{\nu_{1i_1}} \cdots \sum_{\nu_{Mi_M}} \Big[R_{x_{i_1},...,x_{i_M}}^{\alpha-\nu_{1i_1}-\cdots-\nu_{Mi_M}}(\boldsymbol{\tau})_M$$

$$\cdot e^{j2\pi(\nu_{1i_1}\tau_1+\cdots+\nu_{Mi_M}\tau_M)}\Big] \underset{\tau_1}{\otimes} h_{\nu_{1i_1}}(\tau_1) \cdots \underset{\tau_M}{\otimes} h_{\nu_{Mi_M}}(\tau_M), \tag{10}$$

where $\underset{\tau_i}{\otimes}$ denotes convolution with respect to the variable τ_i and the equality has been obtained interchanging the order of integration and summation operations. Note that optional conjugations in the expression of the lag product waveform have been omitted here without loss of generality.

The reduced-dimension version $\overline{R}_{\mathbf{y}}^{\alpha}(\boldsymbol{\tau}')_M$ of $R_{\mathbf{y}}^{\alpha}(\boldsymbol{\tau})_M$ is obtained setting $\tau_M = 0$. Moreover, accounting for (3), the input/output relation in terms of RD-CSCMF's can be written as

$$\overline{S}_{\mathbf{y}}^{\alpha}(\boldsymbol{f}')_M = \sum_{i_1,...,i_M=1}^{N} \sum_{\nu_{1i_1}} \cdots \sum_{\nu_{Mi_M}}$$

$$\overline{S}_{x_{i_1},...,x_{i_M}}^{\alpha-\nu_{1i_1}-\cdots-\nu_{Mi_M}}(\boldsymbol{f}' - [\nu_{1i_1}, ..., \nu_{M-1i_{M-1}}]^{\mathrm{T}})_M$$

$$\cdot H_{\nu_{Mi_M}}(\alpha - \boldsymbol{f}'^{\mathrm{T}}\mathbf{1}) \prod_{m=1}^{M-1} H_{\nu_{mi_m}}(f_m), \tag{11}$$

where $H_{\nu_{mn}}(f_m)$ is the Fourier transform of $h_{\nu_{mn}}(\tau_m)$.

Equations (10) and (11) can be specialized to MIMO LTI systems for which the elements of the impulse response matrix are given by $h_{mn}(t+\tau_m, t) = h_{mn}(\tau_m)$ and, hence, from (9) one has

$$h_{\nu_{mn}}(\tau_m) = h_{mn}(\tau_m)\delta_{\nu_{mn}}, \tag{12}$$

where $\delta_{\nu_{mn}} = 1$ for $\nu_{mn} = 0$ and $\delta_{\nu_{mn}} = 0$ otherwise. In particular, (11) becomes

$$\overline{S}_{\mathbf{y}}^{\alpha}(\boldsymbol{f}')_M = \sum_{i_1,...,i_M=1}^{N} \overline{S}_{x_{i_1},...,x_{i_M}}^{\alpha}(\boldsymbol{f}')_M$$

$$\cdot H_{Mi_M}(\alpha - f'^{\mathrm{T}}\mathbf{1}) \prod_{m=1}^{M-1} H_{mi_m}(f_m). \tag{13}$$

An interesting special class of MIMO linear systems is that of N-SISO LTI systems. In such a case the impulse response matrix is diagonal, that is,

$$h_{mn}(\tau_m) = h_n(\tau_n)\delta_{mn}, \qquad m, n = 1, ..., N. \tag{14}$$

Then, from (13) it follows that

$$\overline{S}^{\alpha}_{\mathbf{y}}(f')_N = \overline{S}^{\alpha}_{\mathbf{x}}(f')_N H_N(\alpha - f'^{\mathrm{T}}\mathbf{1}) \prod_{n=1}^{N-1} H_n(f_n). \tag{15}$$

Such a relation leads to a useful LTI system identification formula. In fact, by letting in (15) $H_n(\cdot) = 1$ for $n = 2, ..., N$ and assuming the input N-dimensional vector $\mathbf{x}(t) \triangleq [x(t), ..., x(t)]^{\mathrm{T}} \triangleq [x(t), \mathbf{x}_0(t)^{\mathrm{T}}]^{\mathrm{T}}$, it results that

$$H_1(f_1) = \frac{\overline{S}^{\alpha}_{y_1\mathbf{x}_0}(f')_N}{\overline{S}^{\alpha}_{\mathbf{x}}(f')_N}. \tag{16}$$

This formula is the generalization to higher-order cyclostationary input signals of the system identification formula stated in [1] with reference to input signals exhibiting second-order cyclostationarity.

Let us consider now the special case of MIMO LAPTV systems characterized by the input/output relation

$$y(t) = \boldsymbol{W}(t)x(t). \tag{17}$$

In such a case $h_{mn}(t + \tau_m, t) = w_{mn}(t)\delta(\tau_m)$, where the elements $w_{mn}(t)$ of the matrix $\boldsymbol{W}(t)$ can be expressed as

$$w_{mn}(t) = \sum_{\nu_{mn}} w_{\nu_{mn}} e^{j2\pi\nu_{mn}t}. \tag{18}$$

Therefore, by substituting $h_{\nu_{mn}}(\tau_m) = w_{\nu_{mn}}\delta(\tau_m)$ into (10), one obtains that

$$R^{\alpha}_{\mathbf{y}}(\tau)_M = \sum_{i_1,...,i_M=1}^{N} \sum_{\nu_{1i_1}} \cdots \sum_{\nu_{Mi_M}} w_{\nu_{1i_1}} \cdots w_{\nu_{Mi_M}} \cdot R^{\alpha-\nu_{1i_1}-\cdots-\nu_{Mi_M}}_{x_{i_1},...,x_{i_M}}(\tau)_M e^{j2\pi(\nu_{1i_1}\tau_1+\cdots+\nu_{Mi_M}\tau_M)}, \tag{19}$$

which leads to

$$\overline{S}^{\alpha}_{\mathbf{y}}(f')_M = \sum_{i_1,...,i_M=1}^{N} \sum_{\nu_{1i_1}} \cdots \sum_{\nu_{Mi_M}} w_{\nu_{1i_1}} \cdots w_{\nu_{Mi_M}} \cdot \overline{S}^{\alpha-\nu_{1i_1}-\cdots-\nu_{Mi_M}}_{x_{i_1},...,x_{i_M}}(f' - [\nu_{1i_1}, ..., \nu_{M-1 i_{M-1}}]^{\mathrm{T}})_M. \tag{20}$$

Moreover, if one assumes that $\boldsymbol{W}(t)$ is a diagonal matrix, that is, an N-SISO system is considered, from (20) it follows that

$$\overline{S}^{\alpha}_{\mathbf{y}}(f')_N = \sum_{\nu_1} \cdots \sum_{\nu_N} w_{\nu_1} \cdots w_{\nu_N} \overline{S}^{\alpha-\nu_1-\cdots-\nu_N}_{x_1,...,x_N}(f' - \nu')_N, \tag{21}$$

where the $w_{\nu_{ii}}$'s have been replaced by the w_{ν_i}'s.

As regards the CTCCF of the M outputs of a MIMO LAPTV system, from (5) and (6), accounting for (10) and observing that the subsets μ_i are disjoint, it follows that

$$C^{\alpha}_{\mathbf{y}}(\tau)_M = \sum_{i_1,...,i_M=1}^{N} \sum_{\nu_{1i_1}} \cdots \sum_{\nu_{Mi_M}} \left[C^{\alpha-\nu_{1i_1}-\cdots-\nu_{Mi_M}}_{x_{i_1},...,x_{i_M}}(\tau)_M e^{j2\pi(\nu_{1i_1}\tau_1+\cdots+\nu_{Mi_M}\tau_M)}\right] \underset{\tau_1}{\otimes} h_{\nu_{1i_1}}(\tau_1) \cdots \underset{\tau_M}{\otimes} h_{\nu_{Mi_M}}(\tau_M). \tag{22}$$

Let us note that the input/output relations in terms of CTCMF's and CTCCF's are the same (compare (22) with (10)). Therefore, the input/output relations in terms of CSCCF's, RD-CTCCF's, and CCP's can be immediately stated by considering the relations in terms of CSCMF's, RD-CTCMF's, and RD-CSCMF's, respectively. Finally, a LTI system identification formula, analogous to (16), can be stated in terms of CCP's:

$$H_1(f_1) = \frac{\overline{P}^{\alpha}_{y_1\mathbf{x}_0}(f')_N}{\overline{P}^{\alpha}_{\mathbf{x}}(f')_N}. \tag{23}$$

4. Applications

We consider here two examples of application of the relations stated in Section 3. The first one considers the problem of the LTI system identification based on input and output noisy measurements; the second one deals with the evaluation of the RD-CSCMF's for some signals of interest in communications.

4.1 LTI system identification

Let us consider a LTI system with impulse response $h(t)$ whose noisy input and output measurements $v(t)$ and $z(t)$ are

$$v(t) = x(t) + n(t), \tag{24}$$
$$z(t) = y(t) + m(t), \tag{25}$$

where $x(t)$ is the exciting signal, $y(t) = x(t) \otimes h(t)$, and $n(t)$ and $m(t)$, which are possibly correlated with each other, model noise and interference and are assumed to be independent (in the fraction-of-time probability sense) of $x(t)$.

By assuming that there exists a cycle frequency β such that the CCP $\overline{P}^{\beta}_{\mathbf{x}}(f')_N$ be not identically zero, from (24) and (25) it follows that

$$\frac{\overline{P}^{\beta}_{z\mathbf{v}_0}(f')_N}{\overline{P}^{\beta}_{\mathbf{v}}(f')_N} = \frac{\overline{P}^{\beta}_{y\mathbf{x}_0}(f')_N + \overline{P}^{\beta}_{m\mathbf{n}_0}(f')_N}{\overline{P}^{\beta}_{\mathbf{x}}(f')_N + \overline{P}^{\beta}_{\mathbf{n}}(f')_N}, \tag{26}$$

where the N-dimensional vectors $\mathbf{n}(t) \triangleq [n(t), ..., n(t)]^{\mathrm{T}} \triangleq [n(t), \mathbf{n}_0(t)^{\mathrm{T}}]^{\mathrm{T}}$ and $\mathbf{v}(t) \triangleq [v(t), ..., v(t)]^{\mathrm{T}} \triangleq [v(t), \mathbf{v}_0(t)^{\mathrm{T}}]^{\mathrm{T}}$ have been defined. Thus, if one assumes that

$$\overline{P}^{\beta}_{\mathbf{n}}(f')_N = \overline{P}^{\beta}_{m\mathbf{n}_0}(f')_N \equiv 0, \tag{27}$$

accounting for (23), it results that

$$\frac{\overline{P}^{\beta}_{z\mathbf{v}_0}(f')_N}{\overline{P}^{\beta}_{\mathbf{v}}(f')_N} = \frac{\overline{P}^{\beta}_{y\mathbf{x}_0}(f')_N}{\overline{P}^{\beta}_{\mathbf{x}}(f')_N} = H(f_1). \tag{28}$$

Consequently, the LTI system can be identified by estimating the CCP's $\overline{P}^{\beta}_{z\mathbf{v}_0}(f')_N$ and $\overline{P}^{\beta}_{\mathbf{v}}(f')_N$ involved in (28).

The system identification problem described by (24) and (25) can also be solved by the formula

$$H(f_1) = \frac{\overline{S}^{\alpha}_{y\mathbf{x}_0}(f')_N}{\overline{S}^{\alpha}_{\mathbf{x}}(f')_N} = \frac{\overline{S}^{\alpha}_{z\mathbf{v}_0}(f')_N}{\overline{S}^{\alpha}_{\mathbf{v}}(f')_N}, \tag{29}$$

which follows from (16) on the assumptions that $x(t)$ exhibits Nth-order cyclostationarity with cycle frequency α and, moreover,

$$\overline{S}^{\gamma}_{\mathbf{x}}(f')_k \equiv 0, \quad \gamma \neq 0; \quad k = 1, ..., N-1, \tag{30}$$

$$\overline{S}^{\alpha}_{m}(f_1)_1 = \overline{S}^{\alpha}_{\mathbf{n}}(f')_k = \overline{S}^{\alpha}_{m\mathbf{n}_0}(f')_k \equiv 0, \quad k = 1, ..., N. \tag{31}$$

It is worthwhile to underline that no problem arises from assumption (30) since the cyclostationarity of lowest order exhibited by $x(t)$ is usually exploited to avoid unnecessary computational complexity. Moreover, assumption (31) on noise and interfering signals is more restrictive than (27).

4.2 Evaluation of higher-order cyclic spectra

Let us consider N time-series $x_{\delta_m}(t)$ each of which is expressed as a product of an impulse train and a time-series, that is,

$$x_{\delta_m}(t) \triangleq x_m(t) \sum_{k=-\infty}^{+\infty} \delta(t - kT_m), \quad m = 1, ..., N. \tag{32}$$

Their joint characterization in terms of RD-CSCMF's can be derived substituting $\nu_m = r_m/T_m$ (with r_m any integer number) and $w_{\nu_m} = 1/T_m$ into (21). It results that

$$\overline{S}^{\alpha}_{\mathbf{x}_\delta}(f')_N = \left(\prod_{m=1}^{N} \frac{1}{T_m}\right) \sum_{\mathbf{r}} \overline{S}^{\alpha - \mathbf{r}^{\mathrm{T}}\mathbf{f}_s}_{x_1, ..., x_N}(f' - \mathbf{r}' \circ \mathbf{f}'_s)_N, \tag{33}$$

where $\mathbf{f}_s \triangleq [1/T_1, ..., 1/T_N]^{\mathrm{T}}$, $\mathbf{r} \triangleq [r_1, ..., r_N]^{\mathrm{T}}$, and $\circ$ denotes the Hadamard matrix product, i.e., $\mathbf{r}' \circ \mathbf{f}'_s \triangleq [r_1/T_1, ..., r_{N-1}/T_{N-1}]^{\mathrm{T}}$.

The RD-CSCMF of N PAM signals

$$\begin{aligned} x_{\mathrm{PAM}_m}(t) &\triangleq \sum_{k=-\infty}^{+\infty} x_m(kT_m) p_m(t - kT_m) \\ &= x_{\delta_m}(t) \otimes p_m(t), \quad m = 1, ..., N, \end{aligned} \tag{34}$$

according to (15), can be written as

$$\overline{S}^{\alpha}_{\mathbf{x}_{\mathrm{PAM}}}(f')_N = P_N(\alpha - f'^{\mathrm{T}}\mathbf{1}) \prod_{m=1}^{N-1} P_m(f_m) \overline{S}^{\alpha}_{\mathbf{x}_\delta}(f')_N, \tag{35}$$

where $\overline{S}^{\alpha}_{\mathbf{x}_\delta}(f')_N$ is given by (33) and $P_m(f)$ denotes the Fourier transform of $p_m(t)$.

Finally, let us consider N quadrature-carrier amplitude-modulation (QAM) time-series

$$\begin{aligned} x_{\mathrm{QAM}_m}(t) &\triangleq c_m(t)\cos(2\pi f_{cm} t + \phi_m) \\ &- s_m(t)\sin(2\pi f_{cm} t + \phi_m), \quad m = 1, ..., N. \end{aligned} \tag{36}$$

These QAM signals can be considered as the outputs of a MIMO LAPTV system excited by $\mathbf{x}(t) \triangleq [x_1(t), ..., x_{2N}(t)]^{\mathrm{T}}$, where

$$x_{2m-1}(t) = c_m(t), \tag{37}$$

$$x_{2m}(t) = s_m(t). \tag{38}$$

The input/output relationship of the system is given by (17), where the nonzero elements of the $N \times 2N$ matrix $W(t)$ are given by

$$w_{m,2m-1}(t) = \cos(2\pi f_{cm} t + \phi_m), \tag{39}$$

$$w_{m,2m}(t) = -\sin(2\pi f_{cm} t + \phi_m). \tag{40}$$

Therefore, from (20) it follows that

$$\begin{aligned} \overline{S}^{\alpha}_{\mathbf{x}_{\mathrm{QAM}}}(f')_N &= \frac{1}{2^N} \sum_{i_1=1}^{2} \cdots \sum_{i_N=2N-1}^{2N} \\ &\sum_{q_{1i_1}=\pm 1} \cdots \sum_{q_{Ni_N}=\pm 1} \left(\prod_{m=1}^{N} e^{jq_{mi_m}[\phi_m + (i_m - 2m + 1)\pi/2]}\right) \\ &\cdot \overline{S}^{\alpha - \mathbf{f}_c^{\mathrm{T}}\mathbf{q}_{i_1,...,i_N}}_{x_{i_1},...,x_{i_N}}(f' - \mathbf{f}'_c \circ \mathbf{q}'_{i_1,...,i_N})_N \end{aligned} \tag{41}$$

where $\mathbf{q}_{i_1,...,i_N} \triangleq [q_{1i_1}, ..., q_{Ni_N}]^{\mathrm{T}}$ and $\mathbf{f}_c \triangleq [f_{c1}, ..., f_{cN}]^{\mathrm{T}}$.

References

[1] W.A.Gardner, *Statistical Spectral Analysis: A Nonprobabilistic Theory.* Prentice Hall, Englewood Cliffs, NJ, 1988.

[2] W.A.Gardner, "Signal interception: a unifying theoretical framework for feature detection," *IEEE Trans. Commun.,* vol. COM-36, pp. 897-906, August 1988.

[3] L.Izzo, L.Paura, and M.Tanda, "Signal interception in non-Gaussian noise," *IEEE Trans. Commun.,* vol. 40, pp. 1030-1037, June 1992.

[4] G.Gelli, L.Izzo, A.Napolitano, and L.Paura, "Multipath-channel identification by an improved Prony algorithm based on spectral correlation measurements," *Signal Processing,* vol. 31, pp. 17-29, March 1993.

[5] B.G.Agee, S.V.Shell, and W.A.Gardner, "Spectral self-coherence restoral: a new approach to blind adaptive signal extraction using antenna arrays," *Proc. of the IEEE,* vol. 78, pp. 753-767, April 1990.

[6] L.Izzo, L.Paura, and G.Poggi, "An interference-tolerant algorithm for localization of cyclostationary-signal sources," *IEEE Trans. Signal Processing,* vol. 40, pp. 1682-1686, July 1992.

[7] W.A.Gardner, "Spectral characterization of N-th order cyclostationarity," in *Proc. of the Fifth ASSP Workshop on Spectrum Estimation and Modeling,* Rochester, NY, October 1990.

[8] W.A.Gardner and C.M.Spooner, "Higher-order cyclostationarity, cyclic cumulants, and cyclic polyspectra," in *Proc. of the International Symposium on Information Theory and its Applications (ISITA'90),* Honolulu, Hawaii, November 1990.

[9] C.M.Spooner, "Theory and application of higher-order cyclostationarity," Ph.D. Dissertation, Dept. of Electrical and Computer Engineering, University of California, Davis, CA, June 1992.

[10] A.V.Dandawate and G.B.Giannakis, "Polyspectral analysis of non-stationary signals: system identification, classification and ambiguity functions," in *Proc. of the International Signal Processing Workshop on Higher Order Statistics,* Chamrousse, F, July 1991.

[11] W.A.Gardner and W.A.Brown, "Fraction-of-time probability for time-series that exhibit cyclostationarity," *Signal Processing,* vol. 23, pp. 273-292, June 1991.