



# ARRAY DETECTION OF WEAK SIGNALS WITH DRIFTING PHASE IN NON-GAUSSIAN NOISE

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## RÉSUMÉ

On considère ici le problème de la détection, au moyen d'un réseau de capteurs, d'un signal sinusoïdal faible affecté de fluctuations de phase et noyé dans un bruit non gaussien. Le modèle utilisé pour les fluctuations de phase est le processus du mouvement brownien. On considère tout d'abord le détecteur localement optimal, puis deux détecteurs sous-optimaux. Les performances des détecteurs sont évaluées et comparées au moyen de la déflection.

## ABSTRACT

The problem of detecting by an array of sensors a weak sinusoid with drifting phase in non-Gaussian noise is addressed. The phase drift is modeled as a Brownian motion process. The locally optimum and two suboptimum detectors are considered. Their performances are evaluated and compared in terms of deflection.

## 1. INTRODUCTION

The problem of detecting a weak sinusoidal acoustic signal with drifting phase has been considered in [1] with reference to the case of additive Gaussian noise. The performance of the quadratic detector, which is locally optimum, has been compared with those of two suboptimum structures of easier implementation. The first one is the standard noncoherent detector, which is optimum for detecting a sinusoidal signal with a random but constant phase (i.e., in the absence of phase drift) embedded in Gaussian noise. The second one, referred to as the *m*th-order noncoherent detector [2], trades off coherent and noncoherent integration of the received waveform to assure satisfactory performances in the presence of significant phase drifts.

Since in underwater acoustics the statistical characteristics of the background noise significantly deviate from the Gaussian ones [3], the conventional systems (i.e., those optimized against Gaussian noise) can exhibit a drastic performance degradation. Therefore, in weak-signal conditions, locally optimum detection structures for non-Gaussian noise have to be considered. Moreover, to reduce the long collect time, which is required to assure reliable detection, an array of sensors is commonly adopted [4,5].

The present paper deals with the problem of detecting, by an array of sensors, a weak sinusoidal signal with drifting phase in the presence of additive non-Gaussian noise. The phase drift of the signal of interest at each array element is modeled as a Brownian motion process. In our observation model, the phase drifts are assumed to be mutually independent and, moreover, the noise samples are assumed to be spatially and temporally independent.

The locally optimum array detector for the problem at hand is first considered. Then, two suboptimum detection

structures are examined. The first one is the locally optimum array detection structure synthesized to detect a sinusoidal signal with a random but constant phase in the presence of non-Gaussian noise. The second one is a version of the *m*th-order noncoherent detector (considered in [1,2]), which is modified to account for the non-Gaussian behavior of the background noise.

The performances of the considered detectors are compared in terms of deflection, which is a useful performance measure in weak-signal conditions. In particular, the order *m* of the *m*th-order noncoherent detector is selected by maximizing the deflection of the decision statistic.

## 2. DETECTION STRUCTURES

The detection problem under consideration can be represented by the hypothesis test

$$\begin{aligned} H_0 : y_p(i) &= n_p(i) \\ H_1 : y_p(i) &= A_p \cos(2\pi\nu_0 i + \theta_p(i) + \phi_p) + n_p(i) \end{aligned} \quad (1)$$

where  $y_p(i)$  and  $n_p(i)$  ( $p = 1, 2, \dots, M$ ;  $i = 1, 2, \dots, N$ ) denote the received signal and the noise (respectively) at the *i*th instant on the *p*th element of a receiving array of *M* elements. The noise samples  $n_p(i)$  are assumed to be independent and identically distributed random variables (RV's) with a first-order probability density function (PDF)  $f(\cdot)$ , zero mean, and unit variance. The signal of interest at the input of the *p*th array element has amplitude  $A_p$ , a known frequency  $\nu_0$ , and a phase  $\theta_p(i) + \phi_p$  where  $\phi_p$  is a random initial phase, uniformly distributed in the interval  $[0, 2\pi]$ , and  $\{\theta_p(i)\}$  is the phase drift process on the *p*th channel, which



is modeled here as a Brownian motion process, i.e.,

$$\theta_p(i) = \sum_{k=1}^i w_p(k) \quad (2)$$

where  $\{w_p(k)\}$  is a zero-mean white Gaussian process with variance  $\sigma_p^2$ . All of the phase drift processes, the RV's  $\phi_p$ , and the noise processes are statistically independent with each other.

Under the previous assumptions the signal  $s_p(i) = A_p \cos(2\pi\nu_0 i + \theta_p(i) + \phi_p)$  is a wide-sense stationary process with autocorrelation function

$$K_{s_p}(q) = \frac{A_p^2}{2} \rho_p^{|q|} \cos(2\pi\nu_0 q) \quad (3)$$

where  $\rho_p \triangleq \exp(-\sigma_p^2/2)$ . Note that  $A_p^2/2$  is the signal-to-noise ratio (SNR) at the input of the  $p$ th array element, which will be denoted by  $Z_p$ . Moreover,  $\rho_p$  is the phase drift parameter, which assumes a unity value in the absence of phase drift.

The sufficient statistic of the locally optimum detector (LOD) for the problem stated in (1) can be written as [6]

$$T^{LOD} = \sum_{p=1}^M Z_p \sum_{i,j=1}^N \rho_p^{|i-j|} \cos(2\pi\nu_0(i-j)) \cdot [g(y_p(i))g(y_p(j)) + \dot{g}(y_p(i))\delta_{ij}] \quad (4)$$

where  $g(y) \triangleq \dot{f}(y)/f(y)$ , the dot denotes the derivative, and  $\delta_{ij}$  is the Kronecker delta.

To reduce the implementation difficulties of the LOD, it is interesting to resort to suboptimum detectors. One such detector is the LOD for the problem stated in (1) when the phase drifts are absent. Its detection statistic is obtained by setting  $\rho_p = 1$  in (4). Since in the particular case of Gaussian noise ( $g(y) = -y$ ) such a detection statistic becomes that of the standard noncoherent detector, the detector under consideration will be referred to as the noncoherent detector (NCD).

The NCD is expected to perform poorly when the signal phases drift significantly over the observation interval. Then, a modified version of such a detector, referred to as the  $m$ th-order noncoherent detector ( $m$ NCD), can be introduced. The  $N$  samples received at each array element are splitted into  $m$  segments of  $K$  samples and, then, the following decision statistic is formed:

$$T^{mNCD} = \sum_{p=1}^M Z_p \sum_{j=1}^m \left\{ \left[ \sum_{i=1}^K g(y_p(i + (j-1)K)) \cos(2\pi\nu_0(i + (j-1)K)) \right]^2 + \left[ \sum_{i=1}^K g(y_p(i + (j-1)K)) \sin(2\pi\nu_0(i + (j-1)K)) \right]^2 + \sum_{i=1}^K \dot{g}(y_p(i + (j-1)K)) \right\} \quad (5)$$

A suitable value of  $m$  can be selected by optimizing some performance measure. In the following the deflection criterion is adopted. We note that in the limiting case of the

heaviest phase-drift conditions (i.e.,  $\rho_p = 0$  for any  $p$ ) the LOD reduces to the  $m$ NCD of order  $m = N$  and, hence, in this case the choice of  $m$  is obvious.

### 3. PERFORMANCE ASSESSMENT

Since the analytical evaluation of the conditional PDF's of the decision variables under both hypotheses is an intractable problem, the detector performances are stated here in terms of deflection.

The deflection of a decision statistic  $T$  is defined by

$$D(T) \triangleq \frac{[E_1(T) - E_0(T)]^2}{VAR_0(T)} \quad (6)$$

where  $E_0(\cdot)$ ,  $E_1(\cdot)$ , and  $VAR_0(\cdot)$  denote the expectations conditioned to  $H_0$  and  $H_1$ , and the variance conditioned to  $H_0$  of the decision variable  $T$ .

The detection statistic that maximizes the deflection is the likelihood ratio, say  $\Lambda$ , whose deflection is given by [7]

$$D(\Lambda) = VAR_0(\Lambda) \quad (7)$$

provided that  $VAR_0(\Lambda) < \infty$ . Such a result can be usefully exploited to evaluate the deflection of the LOD operating in weak-signal conditions. In fact, for weak signals,  $\Lambda \simeq 1 + T^{LOD}/2$  and, then, it results that

$$D(\Lambda) \simeq D(T^{LOD}) \simeq \frac{1}{4} VAR_0(T^{LOD}) = \frac{1}{4} \sum_{p=1}^M VAR_0(T_p^{LOD}) \simeq \sum_{p=1}^M D(T_p^{LOD}) \quad (8)$$

where  $T_p^{LOD}$  denotes the locally optimum detection statistic for the case of a single receiving sensor ( $M = 1$ ) and the assumption of statistically independent noises has been accounted for. Therefore, from (4) and (8) it follows that

$$D(T^{LOD}) = \frac{N}{4} \sum_{p=1}^M Z_p^2 \{ 2E_0^2[g^2(y)]S(N, \rho_p) + VAR_0[g^2(y) + \dot{g}(y)] \} \quad (9)$$

where

$$S(N, \rho_p) \triangleq \sum_{\substack{q=1 \\ q \neq 0}}^{N-1} \left( 1 - \frac{|q|}{N} \right) \rho_p^{2|q|} \cos^2(2\pi\nu_0 q) \quad (10)$$

As regards the deflection of the  $m$ th-order noncoherent detector, it can be shown that, under weak-signal conditions, one has

$$D(T^{mNCD}) = \frac{mK}{4}$$

$$\frac{\left\{ \sum_{p=1}^M Z_p^2 [2E_0^2[g^2(y)]S(K, \sqrt{\rho_p}) + VAR_0[g^2(y) + \dot{g}(y)]] \right\}^2}{[2E_0^2[g^2(y)]S(K, 1) + VAR_0[g^2(y) + \dot{g}(y)]] \sum_{p=1}^M Z_p^2} \quad (11)$$

Finally, the deflection of the NCD is obtained by putting  $m = 1$ , and hence  $K = N$ , in (11).



$c \backslash \rho$	0	0.4	0.6	0.7	0.8	0.9	0.92	0.94	1
1.6	30	30	15	6	3	2	1	1	1
2	30	30	15	6	3	2	1	1	1
2.4	30	30	15	6	3	2	1	1	1
3	30	30	15	10	5	2	1	1	1
4	30	30	15	10	5	2	1	1	1
5	30	30	15	10	6	2	2	1	1
6	30	30	15	10	6	2	2	1	1
7	30	30	15	10	6	2	2	1	1
8	30	30	15	10	6	2	2	1	1

Table 1

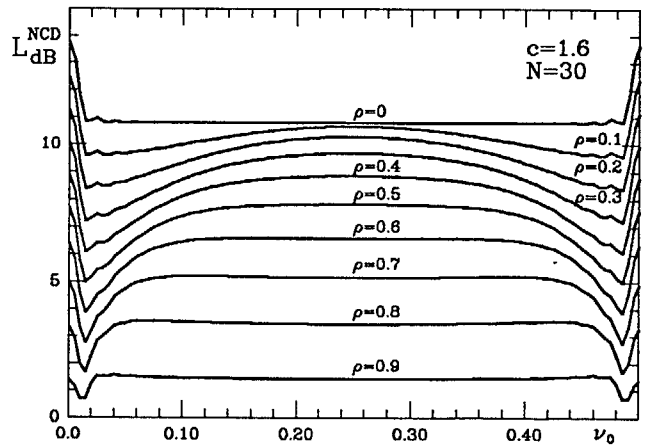


Fig.2a

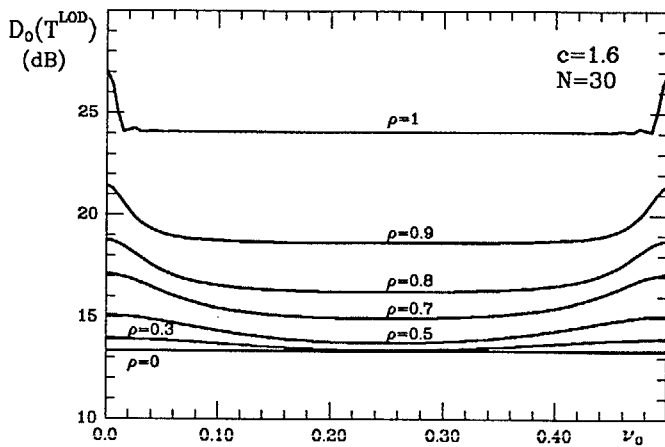


Fig.1a

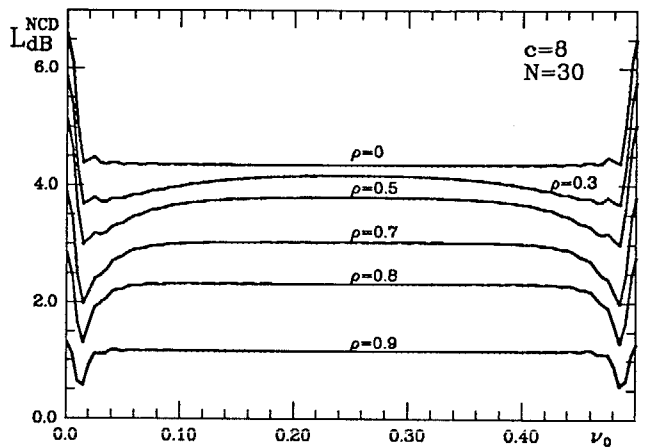


Fig.2b

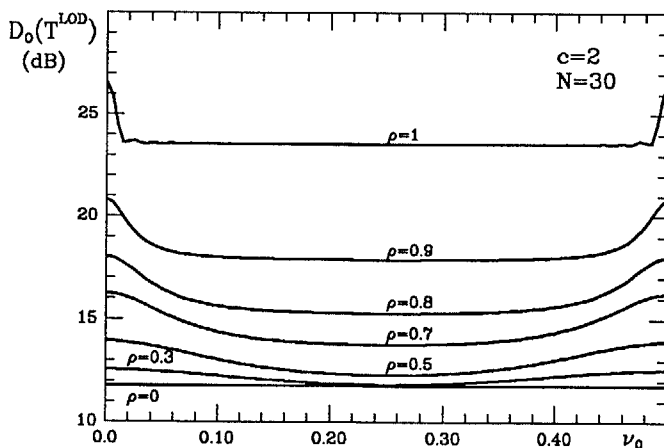


Fig.1b

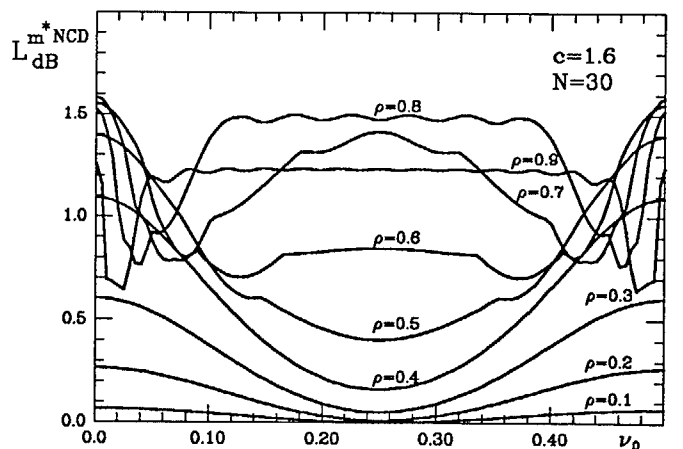


Fig.3a

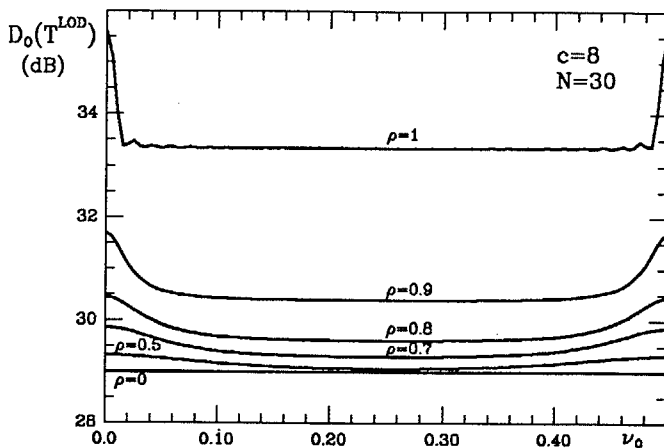


Fig.1c

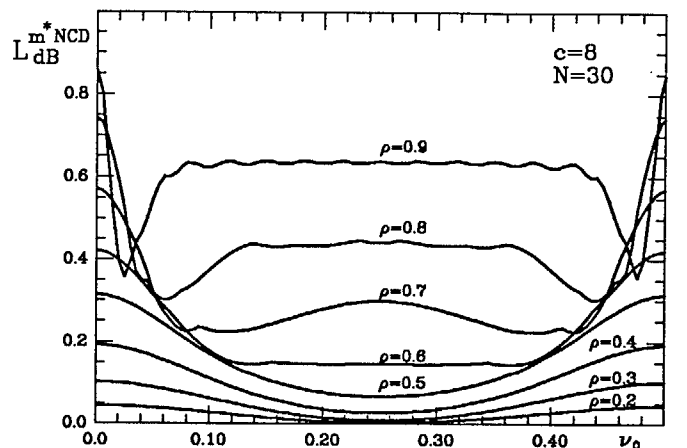


Fig.3b



To assess and compare the performances (in terms of deflection) of the considered detectors, the generalized Gaussian family of distributions is assumed in the following for the first-order statistical characterization of the noise:

$$f(y) = [2\Gamma(1/c)]^{-1} \eta(c)c \exp\{-[\eta(c)|y|^c]\} \quad (12)$$

where

$$\eta(c) \triangleq [\Gamma(3/c)/\Gamma(1/c)]^{1/2} \quad (13)$$

$\Gamma(\cdot)$  is the gamma function,  $c$  is a positive parameter governing the rate of the PDF decay ( $c = 1$  for Laplacian noise,  $c = 2$  for Gaussian noise,  $c \simeq 8$  for nearly uniform noise). Such a PDF family is quite flexible, encompassing a wide class of PDF's and, moreover, models the characteristics of a variety of man-made and natural noise sources [5,8].

Figures 1a-1c report the deflection normalized to the square of the SNR at the input of the detector, say  $D_0(T^{LOD})$ , for the LOD in the case of a single receiving sensor, by assuming (as in the following) a sample size  $N = 30$ . The normalized deflection is plotted as a function of the frequency  $\nu_0$  for several values of the drift parameter  $\rho$  and for  $c = 1.6$  (Fig.1a),  $c = 2$  (Fig.1b), and  $c = 8$  (Fig.1c). It is shown that  $D_0(T^{LOD})$  decreases as  $\rho$  decreases from  $\rho = 1$  (absence of phase drift) to  $\rho = 0$  (white signal case). Moreover, the performance loss (with respect to the case of absence of drift) increases less and less as  $\rho$  approaches zero. As regards the deflection as a function of the frequency  $\nu_0$ , the curves exhibit two peaks for the limiting values  $\nu_0 = 0$  and  $\nu_0 = 0.5$ , except for the case  $\rho = 0$  where  $D_0(T^{LOD})$  is independent of  $\nu_0$  (see also (9) and (10)). Finally, the comparison among the figures shows that for nearly uniform noise ( $c = 8$ ) the deflection assumes the highest values and, moreover, the performance loss due to the drifting phase is the least severe.

Note that, accounting for (8), the results reported in Figs. 1a-1c can also be utilized to evaluate the deflection when a sensor array is considered. In particular, when the phase drift parameters  $\rho_p$  are all equal with each other, the curves of Figs. 1a-1c provide directly  $D(T^{LOD})$  normalized to the summation of the squares of the SNR's. Moreover,  $D(T^{LOD})$  turns out to be proportional to  $M$  when also the SNR's are all equal with each other.

The comparison between the LOD and NCD performances can be easily made on the assumption that the phase drift parameters  $\rho_p$  are all equal with each other, say  $\rho$ . In fact, in this case the ratio  $D(T^{NCD})/D(T^{LOD})$  does not depend on both the number of sensors and the SNR's  $Z_p$  (see (9) and (11)). Figures 2a and 2b present the loss (in decibels)  $L_{dB}^{NCD}$ , in terms of deflection of the NCD with respect to the LOD, as a function of the frequency  $\nu_0$  for some values of  $\rho$  and for  $c = 1.6$  (Fig.2a) and  $c = 8$  (Fig.2b). Note that in the absence of drift there is no performance loss since the NCD is a locally optimum detector. As expected, the loss increases as  $\rho$  decreases, i.e., as the phase drifts become more severe. Moreover, large performance degradations (even about 10dB) can occur. The loss is less severe for nearly uniform noise.

To obtain performances closer to those of the LOD, one can resort to the  $m$ th-order noncoherent detector by exploiting the deflection optimality criterion to select a suitable value of the order  $m$ . By assuming again  $\rho_p = \rho$  for any  $p$ , the value of  $m$  that maximizes the deflection  $D(T^{mNCD})$ , say  $m^*$ , is independent of the number of sensors and SNR's  $Z_p$ . Table 1 reports the value  $m^*$  obtained in correspondence

of some pairs  $(\rho, c)$  for  $\nu_0 = 0.2$ . As expected,  $m^* = N = 30$  for  $\rho = 0$  and  $m^* = 1$  for  $\rho = 1$ , for any value of the noise shape parameter  $c$ .

Figures 3a and 3b present the loss (in decibels)  $L_{dB}^{m^*NCD}$  in terms of deflection of the  $m^*NCD$  with respect to the LOD as a function of  $\nu_0$  for some values of  $\rho$  and for  $c = 1.6$  (Fig.3a) and  $c = 8$  (Fig.3b). Let us note that, according to the previous observation, there is no loss for  $\rho = 0$  and  $\rho = 1$  for any value of  $\nu_0$  and  $c$ . The quite irregular behavior of the loss as a function of  $\nu_0$  is not surprising since the value of  $m^*$  depends on  $\nu_0$  and, then, any curve does not refer to a comparison between two detectors, but rather to the comparison between the class of  $m^*NCD$  and the LOD. The results show that the performance degradation is less than 2dB, regardless of the amount of phase drifts and the value of the noise shape parameter  $c$ .

#### 4. CONCLUSIONS

The problem of detecting by an array of sensors a weak sinusoidal signal with drifting phase in non-Gaussian noise is considered. It is shown that the  $m$ th-order noncoherent detector performs nearly as well as the locally optimum detector when the performance measure is the deflection and the order  $m$  is selected according to the maximization of this performance measure.

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