OPTIMISED RADAR DETECTION IN CORRELATED WEIBULL CLUTTER

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RÉSUMÉ

Dans cet article nous abordons le probleme de la synthèse de systèmes optimaux pour la detection de signaux noyés dans un bruit de Weibull, modelé comme un processus spheriquement invariant. Comme le critère uniformement le plus puissant ne existe pas si la phase du signal est inconnue, nous considérons le deux strategies de Neumann-Pearson et du Rapport de Vraisemblance Generalisé pour eliminer l'incertitude sur le signal. La comparaison, effectuée en termes de compléxité et de performances, montre que la strategie du Rapport de Vraisemblance Generalisé est la plus appropriée pour la detection de signaux inconnues.

ABSTRACT

This paper faces the problem of optimised radar detection in Weibull disturbance, once the compound-Gaussian model is adopted for the received clutter echo. After ascertaining that no Uniformly Most Powerful test exists with respect to the target parameters, we discuss two possible approaches to handle the a-priori uncertainty as to the target signal: the former is the classical Neymann-Pearson criterion, the latter is the Generalised Likelihood Ratio Test. The comparison is carried on in terms of complexity as well as achievable performance, and shows that the Generalised Likelihood Ratio Test is the more attractive means for circumventing the a-priori uncertainty as to the target signal.

1. INTRODUCTION

Optimum detection of radar signals in non-Gaussian noise has received a great deal of attention in the literature so far: in many applications, indeed, such as high-resolution radars and low grazing angles, the actual disturbance amplitude cannot be modeled as a Rayleighian random process. Extensive experimental campaigns, whose results are reported in [1 ÷ 4] for sea clutter, in [3 ÷ 5] for land clutter and in [5, 6] for weather clutter, have demonstrated that two amplitude pdf's (apdf's) achieve the best fit to real data, the Weibull and the K-distribution: nevertheless, an agreement on which one is preferable has not been reached as yet. In any case, experimental data, along with theoretical considerations, indicates that non-Gaussian clutter arises as an effect of a composite scattering mechanism; moreover, its apdf is invariant under Discrete Fourier Transformation (DFT) [2, 5, 6] thus suggesting that the clutter process is inherently a compound-Gaussian one.

Much work has been directed toward the assessment of *conventional* detectors in the presence of non-Gaussian disturbance [7, 8] and the design of new detection schemes in K-distributed clutter [9]. Conventional processors turn out to suffer marked performance degradation under non-Gaussian clutter, while proper optimisation may yield satisfactory detection performance, as demonstrated in [9].

The problem of optimum detection in Weibull clutter is discussed in [10]: a new model for *coherent* Weibull clutter, based on a generalisation to the complex case of the well-known Wiener approach, is introduced and subsequently applied to the design and the assessment of new detection schemes. Some criticism might be raised in that a true optimality cannot be claimed on the proposed schemes. Further, the clutter model is not compatible with the cited invariance of Weibull apdf under DFT, nor does it properly describe the composite scattering mechanism.

As a consequence, the problem of optimised detection in compound-Gaussian noise with Weibull apdf is still open and is the object of this paper. Precisely, in Section 2 we briefly outline the compound-Gaussian model for Weibull clutter; in Section 3, after ascertaining that no Uniformly Most Powerful (UMP) test exists, we thoroughly discuss the possible approaches to design optimised detectors for partially known radar signals. Section 4 is devoted to a comparison between the different strategies in terms of both the achievable performance and complexity and robustness with respect to changing operating conditions.

2. CLUTTER MODEL

As pointed out in the introduction, one of the most credited non-Rayleigh apdf is the Weibull one, namely

$$f_A(r) = abr^{b-1}\exp\left(-ar^b\right) \qquad r \geq 0, \quad a, b > 0 \qquad (1)$$

where b and a are the *shape* and the *scale* parameter, respectively, related to the common variance of the quadrature components as $(1/2)a^{-2/b}\Gamma(1+2/b) = \sigma^2$. The Rayleigh law is subsumed in the family (1) for $b = 2$, while the members corresponding to $b < 2$ exhibit heavier tails than the Rayleigh, thus accounting for clutter spikyness.

In keeping with experimental data, we assume that the *coherent* clutter echo is a compound-Gaussian process, namely that it can be regarded as the product of a complex Gaussian process times a non-negative process, the so-called spiky component [11]. Moreover, the average decorrelation time of such a component is typically much longer than the dwell time, implying that the modulating process degenerates into a random variate. As a consequence, the complex row vector $\mathbf{n}$, whose entries are samples from the baseband equivalent of the clutter process, is a Spherically Invariant Random Vector (SIRV) [11] and can be written as

$$\mathbf{n} = s\mathbf{g} \tag{2}$$

where $\mathbf{g}$ is a complex Gaussian vector and s is the *modulating* variate, with pdf $f(s)$.

Notice that the adopted model ensures the reported invariance of the clutter apdf under Moving Target Indicator (MTI) processing and DFT since, due to representation (2), it is *closed* with respect to linear transformations.

Remarkably, this model allows a complete specification of the clutter process even in the case of correlated observations. More precisely, the $N-$dimensional pdf of a zero-mean complex SIRV $\mathbf{n}$ with Weibull apdf can be cast as

$$f_{\mathbf{n}}(\mathbf{x}) = \mathrm{B}\, h\left(\| \mathbf{x} \|_{\mathbf{M}}\right) \tag{3}$$

where B is a suitable normalisation factor, $\| \mathbf{x} \|_{\mathbf{M}}$ is the norm of $\mathbf{x}$ defined by the definite-positive matrix $\mathbf{M}^{-1}$, with $\mathbf{M}$ the autocovariance matrix of $\mathbf{n}$ and $h(\cdot)$ is defined as

$$h(x) = \int_0^{+\infty} s^{-2N} e^{\left(-\frac{x^2}{2s^2}\right)} f(s) ds = \sum_{k=1}^{N} A_k x^{kb-2N} e^{-ax^b} \tag{4}$$

where

$$A_k = \sum_{m=1}^{k} (-1)^{m+N} \frac{a^k 2^N}{k!} \binom{k}{m} \frac{\Gamma\left(m\frac{b}{2}+1\right)}{\Gamma\left(m\frac{b}{2}+1-N\right)} \tag{5}$$

Notice that for uncorrelated observations $\| \mathbf{x} \|_{\mathbf{M}}$ reduces to the usual Euclidean norm: however, the multivariate pdf (3) does *not* reduce to the product of the marginal pdf's of the clutter samples, implying that uncorrelation is *not* equivalent to independence.

3. OPTIMISED DETECTION IN WEIBULL CLUTTER

The problem of detecting signals embedded in clutter can be stated in terms of the following hypotheses test

$$\begin{cases} H_0: & \mathbf{r} = \mathbf{n} \\ H_1: & \mathbf{r} = \alpha\mathbf{p} + \mathbf{n} \end{cases} \tag{6}$$

where $\mathbf{r}$, $\mathbf{p}$ and $\mathbf{n}$ are $N-$dimensional, complex vectors whose components are samples from the baseband equivalent of the received signal, of the transmitted signal and of the clutter, respectively, while $\alpha = Ae^{j\theta}$ is a complex gain accounting for the channel effect and the target Radar Cross Section (RCS). The quoted closure property of SIRV's allows one to apply the whitening approach to detect signals in correlated disturbance. Therefore, we can limit ourselves to the case of uncorrelated noise (i.e., the vector $\mathbf{n}$ possesses identity covariance matrix) with the understanding that, if the clutter is correlated, $\mathbf{p}$ represents the useful signal at the output of the whitening filter [9].

At first we consider the case of perfectly known signal: the worth of such a model is twofold, namely to ascertain the existence of a UMP test and to establish an upper bound for the achievable performance in the presence of target with unknown parameters. In this case the solution to the detection problem is straightforward and amounts to implementing the Likelihood Ratio Test (LRT):

$$\frac{h(\| \mathbf{r} - Ae^{j\theta}\mathbf{p} \|)}{h(\| \mathbf{r} \|)} \underset{H_0}{\overset{H_1}{\gtrless}} T \tag{7}$$

The block diagram of the optimum receiver implementing the test (7) is depicted in figure 1a: it coincides with the conventional one, but for the presence of a Zero-Memory Non-Linearity (ZMNL), aimed at warping the norm of the received echo and its distance from the useful signal.

Since the Left Hand Side (LHS) of equation (7) is easily seen not to admit a sufficient statistic independent of the parameters of the useful signal, then no UMP test exists.

Consequently, design of optimised processors in Weibull clutter requires a suitable strategy in order to handle the a-priori uncertainty as to the useful signal. In principle, two different criteria can be adopted. The former, the Neymann-Pearson criterion, which is by far the most common, requires working out a suitable statistical model for the wanted target echo, namely assigning a fluctuation law for the complex gain α; accordingly, the LRT is:

$$\frac{\overline{h(\| \mathbf{r} - Ae^{j\theta}\mathbf{p} \|)}}{h(\| \mathbf{r} \|)} \underset{H_0}{\overset{H_1}{\gtrless}} T \tag{8}$$

where the bar denotes the expectation over the random parameter α. This obviously leads to an *optimum* detector, in the sense that its detection performance is unbeatable by that of any other detector operating under the same signal-parameters fluctuation instances. Unfortunately, in non-Gaussian environment, the Neymann-Pearson strategy usually leads to a detector that cannot be implemented, but only approximated. Moreover its structure depends on the fluctuation law of the signal parameters, and its performance may suffer remarkable degradation from mismatching between the assumed and the actual target fluctuation law.

A different optimisation strategy relies on the so-called Generalised Likelihood Ratio Test (GLRT). This strategy amounts to considering α as an unknown complex parameter rather than as a random variable. Accordingly, its Maximum Likelihood (ML) estimate is substituted into the LRT in place of α. This is tantamount to implementing the test:

$$\max_{\alpha} \frac{h(\| \mathbf{r} - Ae^{j\theta}\mathbf{p} \|)}{h(\| \mathbf{r} \|)} \underset{H_0}{\overset{H_1}{\gtrless}} T \tag{9}$$

namely to maximizing the likelihood ratio with respect to α and comparing such a maximum to a suitable threshold. It is worth noticing here that this test is *not* optimum, namely that it does *not* achieve maximum power for a given false alarm rate. On the other hand, the relevant properties of the ML estimates and, in particular, their consistency ensure that for increasingly high number of the integrated samples the performance of the GLRT should approach the perfect measurement bound, i.e. the performance of the optimum detector for completely known target signal. Moreover, the GLRT is inherently a distribution-free test, namely it leads to a detector whose structure does not depend on the target model being in force (the detector is canonical).

The previous discussion is intended to show that it is not possible to decide a-priori which strategy is preferable, even if the GLRT approach sounds somewhat appealing for radar applications. A final decision, however, cannot but be made a-posteriori, namely after investigating whether the loss suffered by the GLRT with respect to the NP detector is negligible or not. To this end, we considered the situation where the target phase only is either unknown or fluctuating, whilst its amplitude is a-priori known: this model is often referred to as *steady target* in the literature.

4. DETECTION OF STEADY TARGETS

The detection of a steady target, according to the Neymann-Pearson criterion, amounts to implementing the LRT (8), with the understanding that the bar denotes now the expectation over the random phase θ, assumed uniformly distributed in $(0, 2\pi)$. Unfortunately, the integral in the numerator of the LHS of (8) cannot be expressed in a closed form. Thus a practical implementation of this test requires the quantization of the phase, which is allowed to take on the values $\theta_k = 2\pi k/M$, $k = 0, \ldots, M-1$ with one and the same probability $1/M$. The corresponding detector implements the test:

$$\frac{\frac{1}{M}\sum_{k=1}^{M} h(\| \mathbf{r} - Ae^{j\theta_k}\mathbf{p} \|)}{h(\| \mathbf{r} \|)} \underset{H_0}{\overset{H_1}{\gtrless}} T \qquad (10)$$

and is depicted in figure 1b. Such a receiver is asymptotically optimum since, as $M \to \infty$, its performance approaches that of the optimum test (8), and will be referred to as Discrete NP (DNP) detector in the following, as it is the optimum NP detector once the phase is modelled as a *discrete* variate. Notice that this receiver can be regarded as a set of M parallel NP detectors for known signal, the k-th being matched to the phase θ_k.

The GLRT for detecting a steady target in Weibull clutter, instead, implements the test (9), where now the maximization is over the phase θ. Since the function (4) is a decreasing function for non negative arguments, the LHS of (9) is maximum as the squared norm

$$\| \mathbf{r} - Ae^{j\theta}\mathbf{p} \|^2 = \| \mathbf{r}_\perp \|^2 + \left| \frac{\mathbf{r}\cdot\mathbf{p}}{\| \mathbf{p} \|} - Ae^{j\theta} \| \mathbf{p} \| \right|^2 \qquad (11)$$

is minimum, with $\mathbf{r}_\perp$ the component of the received vector orthogonal to the direction of the transmitted signal. As a consequence, the ML estimate of the signal phase, is [9] $\widehat{\theta} = \angle \mathbf{r}\cdot\mathbf{p}$, namely is the phase at the output of a filter matched to the useful signal. Thus the GLRT is

$$\frac{h\left(\sqrt{\| \mathbf{r} \|^2 + A^2 \| \mathbf{p} \|^2 - 2A|\mathbf{r}\cdot\mathbf{p}|}\right)}{h(\| \mathbf{r} \|)} \underset{H_0}{\overset{H_1}{\gtrless}} T \qquad (12)$$

The corresponding receiving structure is depicted in figure 1c and coincides with the receiver for perfectly known signal, but for the presence of the signal estimator.

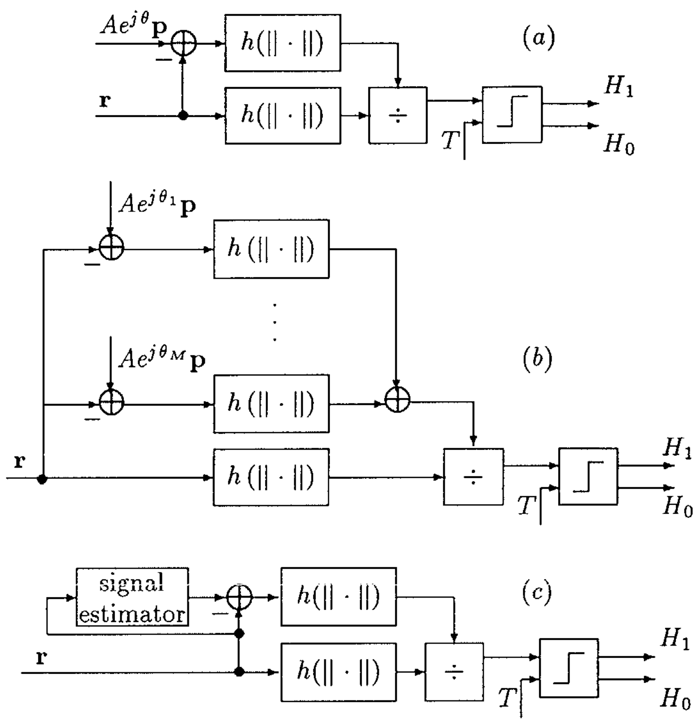

Figure 1: Optimised detectors for coherent pulse trains embedded in correlated Weibull clutter: LRT detector for signals with known parameters (a), DNP detector for steady targets (b), and ML detector for signals with unknown deterministic phase (c).

A direct comparison between these schemes highlights that the ML detector exhibits a much simpler structure than the NP detector, whose complexity linearly increases with the number M of parallel channels: on the other hand, M is to be high enough in order to ensure an acceptable loss with respect to the test (8). Moreover, for at least one value of the clutter shape parameter, i.e. as $b = 2$ and the clutter exhibits Rayleigh apdf, the NP and the ML detector coincide with the classical envelope detector, while the DNP achieves near-optimum performance only if M is high enough (in particular, $M \geq 8$): this fact is clearly demonstrated in figure 2, where the common performance of the NP and the ML detector subject to Gaussian disturbance are contrasted with those of several DNP detectors.

In order to investigate the behaviour of the above detectors under non-Rayleigh clutter, we refer to figure 3, where the performance of the NP detector is shown, along with that of the ML and of the DNP for several M's: for comparison purposes, the perfect measurement bound is reported also. This figure, as well as the previous one, was generated assuming the target phase uniformly distributed in $(0, 2\pi)$, $N = 4$ integrated pulses and Probability of false alarm (P_{fa})

equal to 10^{-4}: it is worth noticing here, however, that the actual phase of the input useful signal does not affect the performance of either the NP or the ML detector; for the DNP detector, instead, such a parameter turns out to be inherently influential.

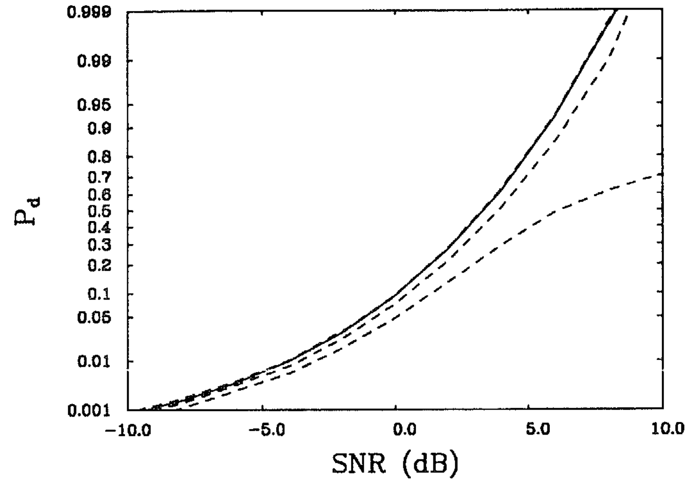

Figure 2: Performance of NP (ML, ——) and DNP ($M = 2, 4, 8, 16$, − − −) detectors for $N = 4$ integrated pulses, $P_{fa} = 10^{-4}$, and $b = 2$ (SNR$= \frac{A^2\|\mathbf{p}\|^2}{2\sigma^2}$).

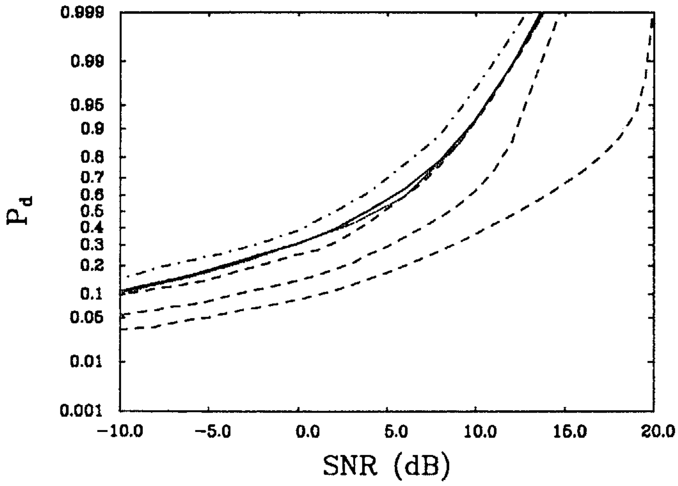

Figure 3: Perfect measurement bound (— · —) and performance of NP (——), DNP (− − −, $M = 2, 4, 8, 16$), and ML (······) detectors for $N = 4$ integrated pulses, $P_{fa} = 10^{-4}$, and $b = 1$ (SNR$= \frac{A^2\|\mathbf{p}\|^2}{2\sigma^2}$).

More generally, in regard to the ML receiver, it can be shown that, since the test statistic depends on the norm of the received signal and on the distance between the received and the *estimated* useful signal, its performance depends on the energy $\| A\mathbf{p} \|^2$ of the wanted target echo, but is otherwise independent of the transmitted waveform and of the phase θ.

Figure 3 highlights that the NP detector is very tightly upperbounded by the perfect measurement bound and lowerbounded by the ML receiver, implying that the GLRT strategy entails a negligible loss with respect to the maximum-power test. Additionally, in order that the parallel-channels implementation (10) do not result into significant detection loss, M should be in the order of 16. Comparing figure 2 with figure 3 suggests that the spikier the clutter apdf, the higher the minimum number of parallel channels ensuring substantial equivalence between the DNP and the NP detector.

Overall, we can conclude that resorting to the GLRT strategy proves advantageous, since it allows one to trade a small additional loss for reduced complexity as well as robustness with respect to the target phase. As a consequence, such an approach suggests itself also when designing optimised detectors for signals with unknown amplitude and phase.

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