OPTIMAL SPATIAL SIGNAL PROCESSING IN NONSTATIONARY WAVEGUIDES

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RÉSUMÉ

Dans l'article ci-dessous nous utilisons un modèle général du signal à modes multiples qui se compose des modes du spectre discontinu avec des intercorrélations arbitraires de l'amplitude pour apprécier l'influence des corrélations de modes sur la caractéristique de la détection du processeur optimum de l'antenne. Cette méthode est basée sur la décomposition orthogonale de Karhunen-Loeve dans l'espace de mode. Nous présentons quelques résultats principaux pour l'antenne horizontale dans le canal d'eau sonore.

ABSTRACT

In this paper, a general model of the multimode signal consisting of the discrete spectrum modes with arbitrary amplitude covatiances is exploited to examine the modal covariance effects on the detection performance of optimal array processing. The approach proposed is based on the orthogonal Karhunen-Loeve expansion in mode space. Some significant results of computer simulation are presented for the horizontal array processing in a shallow-water sound channel.

1. INTRODUCTION

Various aspects of optimal array signal processing in multimode environments, underwater sound channel included, are of particular interest concerned with the source detection/localization problems [1–5]. The basic assumption usually used on the desired signal is the perfectly coherent superposition of spatial harmonics (normal modes) on the array leading to matched-field processing (MFP) [1, 2]. In this paper, it is assumed that the signal and noise background are both partially-coherent multimode fields. The problem of interest is to examine the optimal processor performance sensitivity to the random effects of long-range signal propagation in nonstationary waveguides.

We will address the problem by brief considering the signal-to-noise ratio (SNR) gain for the optimal linear (LP) and quadratic (QP) array processors, emphasizing the weak signal detection problem. The orthogonal Karhunen-Loeve expansion (KLE) in mode space will be used as a general technique for the optimal processor analysis. The computer simulation results will be presented for the horizontal array in a shallow-water sound channel using an exponential model for the signal modal covariances.

2. PRELIMINARIES

It is assumed that the signals from an N-element array are narrow-band N-dimensional data vectors. Furthermore, the desired signal (vector $\mathbf{s}$) and noise background (vector $\mathbf{n}$) are both zero-mean, mutually uncorrelated, Gaussian random processes, and the signal is assumed to be relatively weak. Therefore, detection performance is characterized by the small-signal deflection q of the detection statistic d, or the generalized SNR, defined by

$$q = \frac{< d(\mathbf{s}+\mathbf{n}) > - < d(\mathbf{n}) >}{(< d(\mathbf{n})^2 > - < d(\mathbf{n}) >^2)^{1/2}}, \qquad (1)$$

where $< >$ denotes time averaging.

For a waveguide supporting M normal modes, we consider the data vector $\mathbf{x}$, $\mathbf{x} = \mathbf{s} + \mathbf{n}$, in the normal mode representation [2, 3]:

$$\mathbf{x} = \sum_{m=1}^{M} a_m \mathbf{u}_m + \sum_{m=1}^{M} b_m \mathbf{u}_m + \mathbf{n}_0. \qquad (2)$$

Here $\mathbf{u}_m$ is the deterministic vector of the mth mode shape on the array (the modal vector); a_m and b_m are the random mode amplitudes of the signal and noise interference, respectively; and $\mathbf{n}_0$ is the spatially white (nonmodal) noise vector. We are concerned here with the case of two-dimensional waveguides and use only one modal index (m) for decomposition. Generally speaking, the term "mode" corresponds to some regular spatial shape on the array, characterizing multimode/multipath propagation.

For $\mathbf{x}$ given by Eq. (2), the $(N \times N)$ spatial covariance matrix $\mathbf{M}_x$, $\mathbf{M}_x = < \mathbf{x}^* \mathbf{x}^T >$, is expressed by [2, 3]

$$\mathbf{M}_x = \mathbf{U}^* \mathbf{R}_s \mathbf{U}^T + \mathbf{U}^* \mathbf{R}_n \mathbf{U}^T + \mathbf{I}, \qquad (3)$$

where $\mathbf{U}$ is the $(N \times M)$ modal matrix ($U_{nm} = u_m(n)$); $\mathbf{R}_s = < \mathbf{a}^* \mathbf{a}^T >$ and $\mathbf{R}_n = < \mathbf{b}^* \mathbf{b}^T >$ are the respective $(M \times M)$ modal covariance matrices; $\mathbf{I}$ is the identity matrix (all the modal covariances are normalized to the white noise power); and the typescript ^{T} and asterisk denote transpose and complex conjugate, respectively. It should be emphasized that according to the model (2), (3), the signal and interference are carried by the same set of modes. However, the differences in its modal spectra and modal covariances can be used for the array processor optimization. The differences pointed out depend on both the mode excitation by sources and propagation.

Similarly to Eq. (2), the modal spectrum $\mathbf{v}$ of the array weight vector $\mathbf{w}$ is defined by decomposition [3]:

$$\mathbf{w} = \sum_{m=1}^{M} v_m \mathbf{u}_m^* = \mathbf{U}^* \mathbf{v}. \qquad (4)$$

In addition to the modal spectrum, we define the array modal pattern $\mathbf{g}$ as $\mathbf{g} = \mathbf{U}^T \mathbf{w}$ by analogy with conventional plane-wave beamforming (PWBF). Clearly, the characteristics defined are intrinsically interrelated: $\mathbf{g} = \mathbf{Q}\mathbf{v}$, where the $(M \times M)$ matrix $\mathbf{Q} = \mathbf{U}^T \mathbf{U}^*$.

Finally, the SNR gain G is defined as the deflection (1) normalized to input SNR $q_0 = \mathrm{Tr}(\mathbf{M}_s)/\mathrm{Tr}(\mathbf{M}_n)$: $G = q/q_0$ [5, 6].

3. MODAL KARHUNEN–LOEVE EXPANSION

The first step is to outline the orthogonal expansions of multimode signal. The well-known KLE of the signal in the basis of eigenvectors $(\lambda_p, \mathbf{m}_p)$ of its spatial covariance matrix $\mathbf{M}_s$ was shown [3] to correspond to the modal spectrum decomposition in the basis determined by the following eigenvalue-eigenvector problem:

$$\lambda_p \mathbf{c}_p = \mathbf{R}_s \mathbf{Q} \mathbf{c}_p, \quad p = 1, \ldots, r. \qquad (5)$$

Here the $(M \times 1)$ eigenvectors $\mathbf{c}_p$ are the modal spectra of the $(N \times 1)$ spatial eigenvectors $\mathbf{m}_p$ (i.e. $\mathbf{m}_p = \mathbf{U}^* \mathbf{c}_p$); the eigenvalues λ_p are ordered and normalized:

$$\lambda_1 \geq \lambda_2 \geq \ldots \lambda_r > 0, \quad \sum_{p=1}^{N} \lambda_p = 1, \qquad (6)$$

and $r = \mathrm{rank}(\mathbf{R}_s \mathbf{Q}) \leq \min\{N, M\}$. According to Eqs. (5), (6), the partially coherent multimode signal $(\mathbf{s}, \mathbf{a})$ is the incoherent superposition of the orthogonal coherent eigencomponents $\{\lambda_p, \mathbf{m}_p, \mathbf{c}_p\}_{p=1}^{r}$ with the intensities $\{\lambda_p\}$, spatial shapes $\{\mathbf{m}_p\}$ and modal spectra $\{\mathbf{c}_p\}$. The decomposition presented is the KLE generalized for multimode signal, or the modal KLE. The number r of eigencomponents is considerable, $r \sim M$, if the signal-carrying modes are weakly correlated and the array length is sufficient for their spatial resolution. This case corresponds to coherence-degraded signal, when the coherence length $N_c \ll N$, and the optimal array processor is well known to be substantially complicated [3–6].

The modal KLE is a proper technique for the multimode signal analysis and the detection performance evaluation in dependence on the model of propagation. An obvious advantage of such a technique consists in a possibility of effective exploiting the close relation of spatial signal coherence to its modal covariances to examine the array processor performance/complexity.

4. BASIC EQUATIONS

The second step is to consider the basic equations of LP/QP optimization for the signal model (2), (3), including MFP as a special case.

A general equation of optimal LP beamforming in mode space is given as the M-dimensional eigenvalue-eigenvector problem [3]:

$$q_p \mathbf{v}_p = (\mathbf{I} + \mathbf{R}_n \mathbf{Q})^{-1} \mathbf{R}_s \mathbf{Q} \mathbf{v}_p, \; p = 1, \ldots, r. \quad (7)$$

The largest eigenvalue q_1 is the optimal SNR, and the corresponding eigenvector $\mathbf{v}_1$ determines the optimal modal filter.

For the perfectly coherent signal, the matrix $\mathbf{R}_s$ is dyad, and Eq. (7) reduces to Wiener filtering in mode space:

$$\mathbf{v}_{opt} = \mathbf{v}_0 - \mathbf{B}\mathbf{Q}\mathbf{v}_0, \quad (8)$$

$$q_{opt} = \mathbf{v}_0^H \mathbf{Q} \mathbf{v}_0 - \mathbf{v}_0^H \mathbf{Q}\mathbf{B}\mathbf{Q}\mathbf{v}_0. \quad (9)$$

Here the vector $\mathbf{v}_0 = \mathbf{a}^*$ being the MFP spectrum, the matrix $\mathbf{B} = (\mathbf{R}_n^{-1} + \mathbf{Q})^{-1}$, and the typescript ^{H} denotes conjugate transpose. Assuming that the noise field is incoherent in mode space (i.e., $\mathbf{R}_n = \text{diag}(\sigma_m)$, where σ_m is the modal noise intensity), the optimal filter (8) can be easy written in the scalar form:

$$v_{opt}(m) = v_0(m) - \sigma_m \sum_{q=1}^{M} Q_{mq} v_{opt}(q). \quad (10)$$

Therefore, the most noised modes have the lower weight coefficients in the decomposition (4), and the optimal LP "mismatches" to the signal for effective modal noise prewhitening. In other words, the optimal LP forms deep nulls in the "directions" of the interference-carrying modes. Using the modal pattern $\mathbf{g}$, the last conclusion is expressed by

$$g_{opt}(m) = g_0(m) - \sum_{q=1}^{M} \sigma_q Q_{mq} g_{opt}(q), \quad (11)$$

where $g_0(m)$ are the entries of the "reference" pattern $\mathbf{g}_0 = \mathbf{U}^T \mathbf{s}^*$.

Thus Eqs. (8)–(11) generalize the well-known optimal PWBF (rejection of coherent interference and look-direction bearing) for waveguide propagation.

The optimal QP in mode space is expressed by the same general formula "modal interference prewhitening plus signal mode filtering" but in a more complicated fashion. For the spatially white noise case, the optimal processing lies in the synthesis of the $(N \times r)$ matrix filter $\mathbf{W}$, which is determined by $\mathbf{W}\mathbf{W}^H = \mathbf{M}_s$ [3–6]. Such a processor matches to all spatial signal eigencomponents, and incoherently combines the partial channel outputs for the detection statistic maximization. Since spatial filtering in the basis $\{\lambda_p, \mathbf{m}_p\}$ corresponds to modal filtering in the basis $\{\lambda_p, \mathbf{c}_p\}$ (5), each partial channel ("array") can be considered as the modal filter with the modal pattern $\mathbf{g}_p = \mathbf{U}^T \mathbf{m}_p = \mathbf{Q}\mathbf{c}_p$. Therefore, the optimal QP is obtained by incoherent λ_p-weighted combination of the modal filters matched to the eigencomponents (5), while the optimal LP matches to the most powerful component $(\lambda_1, \mathbf{m}_1, \mathbf{c}_1)$: $G_{QP} = \left(\sum_{p=1}^{r} \lambda_p^2\right) N$, and $G_{LP} = \lambda_1 N$.

Generally, the modal noise leads to spatial coherence of the interference. Therefore, the QP channels need to be corrected in accordance with Eqs. (8)–(11):

$$\mathbf{v}_p = \mathbf{c}_p - \mathbf{B}\mathbf{Q}\mathbf{c}_p, \quad q_{opt} = \left\{ \sum_{p=1}^{r} q_p^2 \right\}^{1/2}, \quad (12)$$

where the values q_p are determined from Eq. (7).

Thus the analysis of spatial signal processing in mode space requires the data on the modal structure and covariances of the received fields. For a relatively small number of modes (for $M \ll N$), the approach presented reduces the problem dimension. Moreover, Eqs. (7)–(12) are effective for *a priori* evaluation of propagation effects on the array performance by simulating the modal spectra and covariances, and the array arrangement in a waveguide for arbitrary numbers M and N. Two intrinsic factors, the modal covariances and the mode orthogonality, are shown to affect mutually the optimal beamforming and the detection performance. Moreover, the modal covariance effects increase with the spatial mode resolution.

5. SIMULATION RESULTS

In this paper, the modal beamforming is simulated for the linear horizontal $\lambda/2$-array in an isovelocity shallow-water sound channel with perfectly rigid bottom and free surface. In such a formulation the modal vectors $\mathbf{u}_m$ (2) are the plane-wave vectors, and the matrix $\mathbf{Q}$ entries coincide with the conventional beampattern factors of the phased array [5]. Nonequidistant spectrum of the isovelocity channel wavenumbers leads to considerable difference (by $\sim M^{3/2}$ times) in spatial "density" of the lower ($m \sim 1$) and the higher ($m \sim M$) modes.

For the signal modal correlations an exponential model is used: $R_{mn} = \exp(-|m-n|/\Delta)$, where the parameter Δ denotes the correlation scale, or the coherence "length" in mode space. As a physical foundation, the stochastic phase modulation of the mode amplitudes in random-inhomogeneous waveguides is considered.

For the purpose of evaluating the signal coherence effects, the simulation is confined to the situation of spatially white noise.

Figure 1 plots the SNR gain loss G/N as a function of the array length for the parameters: $M = 16$, $\Delta = 0.1$ (uncorrelated modes, ——), $\Delta = 10$ (- - -), and $\sin\theta = 0.5$ (θ denotes the angle of arrival).

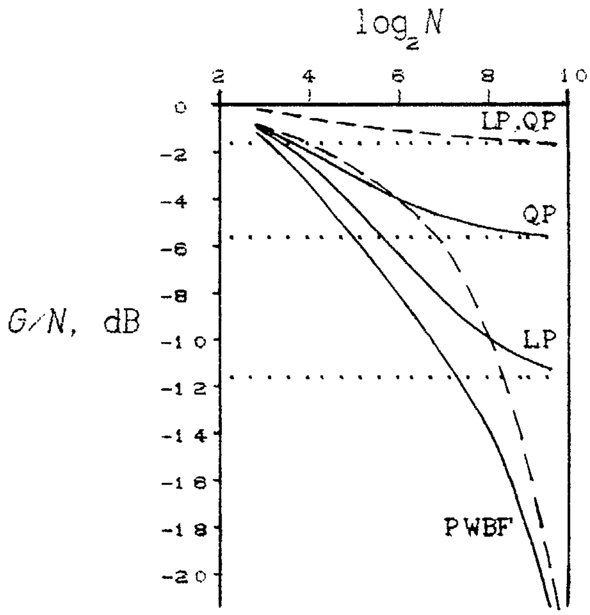

FIG. 1. SNR gain degradation as a function of array length.

A considerable degradation of the LP performance, as can be easy seen, appears only under the conditions of weakly correlated modes and with sufficient mode resolution (increasing with $N\sin\theta$). The optimal QP reduces the gain degradation at a cost of forming additional modal filters and incoherent combining their outputs. The loss "saturation" is determined by the asymptotic eigenvalues (6) for perfect mode resolution (when $\mathbf{Q} \sim \mathbf{I}$). The PWBF performance plotted for comparison (the steering angle is equal to θ) is shown to be extremely sensitive to the multimode propagation.

Figure 2 illustrates the modal spectra $|v_p(m)|$ of five "largest" QP filters for the parameters: $N = 64$, $M = 16$, $\Delta = 0.1$. The pronounced feature is the signal eigencomponents are formed by the different groups of modes. Therefore, the optimal QP channels essentially differ in their modal spectra and patterns. This fact is important from a simplified suboptimal approach point of view. In particular, the optimal LP is the filter of the lowest-order modes.

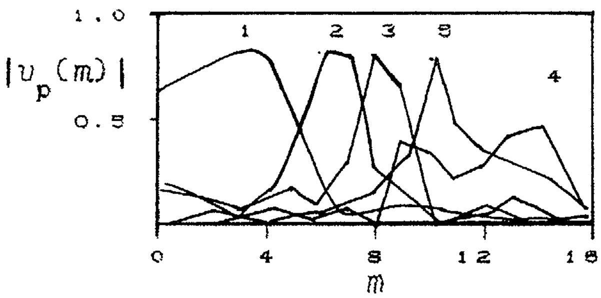

FIG. 2. Modal spectra of the optimal QP channels.

Thus the simulation results exhibit distinctly the array performance sensitivity to the signal modal covariances.

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