EFFECTS OF FAST RANDOM PHASE ERRORS OF WAVEFRONTS ON THE PERFORMANCE OF DIRECTION FINDING

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RÉSUMÉ

L'estimation de direction d'arrivee (en anglais direction of arrival, DOA) utilisant des algorithmes bases sur les valeurs propers necessitent une caracterisation exacte du reseau en termes de geometrie, de gain et de phase des capteurs. En pratique, toutefois, la phase des capteurs different de leurs valeurs nominales du fait d'erreurs de mesure, de changements dans l'environnment et du mauvais placement des capteurs. Nous analysons dans cet article l'erreur quadratique moyenne (MSE) sur les estimees de DOA dans le cas de perturbations de la phase et du gain des capteurs. Tout d'abord nous developpons unc analysc valable pour des sources arbitrairement correless.

ABSTRACT

Direction of arrival (DOA) estimation using adaptive beamforming require exact characterisation of the array in terms of its geometry and sensor gain and phase. In practice, however, the sensor gain and phase differ from nominal values due to measurement errors, changes in the surrounding environment and sensor fluctuations. In this paper, we analyse the mean-square error in the DOA estimates under fast phase perturbations. We develop the analysis for arbitrarily correlated sources.

1. INTRODUCTION

Use of the adaptive beamforming in bearing estimation of closely spaced radiating sources has become very popular due to their superresolution properties [1 – 4]. It has been shown that this method is capable of resolving two independent sources which are separated by an angle which is on the order of one standard beamwidth (sbw) and sometimes less. The performance of this method is severely degraded, however, when coherent, or highly correlated signals are present [1,2,5]. Becides that fluctuations of wavefronts of received signals and element displacement from a nominal location may cause an error in the estimation of the direction of the signals. Effects of random displacement of the array elements has been considered in the literature dealing with the interference cancellation problem [6 – 8]. The purpose of this paper is to obtain analytical estimates of the potential capability for adaptive beamforming in the presence of correlation among the sources and is to study the effects of wavefronts perturbation on the performance of a direction finder. Our resolution analysis is based on the assumption that the covariance matrix of the received signal will be determined exactly by averaging over an infinite period of time, thus enabling us to obtain an estimate of the limiting ability of resolution.

2. SIGNAL MODEL

We consider a passive array, having N sensors, receiving stationary random narrow-band signals emanating from correlated point sources. The received signals are known to be embedded in spatially white noise with unknown variance. Noise is assumed to be statistically independent of the signals. We shall be limited by the assumptions that the array is linear, its sensors are omnidirectional, and received signals have plane wavefronts. Following [2], the MLM adaptive beamforming algorithm involves evaluation of the functional

$$P(\varphi) = (S^{+}\mathbf{R}^{-1}S)^{-1}, \qquad (1)$$

where $\mathbf{R}$ is the covariance matrix and S is direction-of-look vector. Here, we are interested in the two-source case. The covariance matrix of the received signal can be expressed as

$$\begin{aligned}\mathbf{R} \; = \; & \sigma_0^2\mathbf{I} + \sigma_1^2 S_1 S_1^{+} + \sigma_2^2 S_2 S_2^{+} + \sigma_1\sigma_2 \\ & \{\rho S_1 S_2^{+} + S_2 S_1^{+}\rho^{*}\},\end{aligned} \qquad (2)$$

where σ_0^2 is the noise power, σ_p^2 is variance of p-th signal, and S_p is direction vector of p-th signal. S_i is given by

$$S_i = [1, \exp\{j\,u_i\}, \ldots, \exp\{j(N-1)u_i\}]^T, \; i = 1, 2,$$

where (T) denotes transposition, $u_i = (2\pi\delta/\lambda)\sin\varphi_i$, λ is the wavelength, δ is the interelement distance, φ_i is direction of arrival. The coefficient of correlation between signals is defined as follows $\rho = \mid\rho\mid\exp(j\varphi)$, where $\mid\rho\mid$ and φ represent the modulus and argument of the correlation factor. The φ is equal to the constant phase shift between sighals at the coordinate origin. For simplicity, let's place the origin of the coordinates at the geometric center of the antenna. Following [2], we will determine the inverse covariance matrix

$$\begin{aligned}\mathbf{R}^{-1} \; & = \{\mathbf{I} - (1/\alpha)[\nu_1(1 + N\nu_2(1- \mid\rho\mid^2)S_1S_1^{+} - \\ & (N\nu_1\nu_2(1- \mid\rho\mid^2)\mid f(\Delta u)\mid \\ & -\bar{\nu}_1^{-}\bar{\nu}_2\mid\rho\mid\exp(j\varphi))S_1S_2^{+} \\ & -(N\nu_1\nu_2(1- \mid\rho\mid^2)\mid f(\Delta u)\mid -\bar{\nu}_1^{-}\bar{\nu}_2\mid\rho\mid \\ & \exp(-j\varphi))S_2S_1^{+} + +\nu_2(1 + N\nu 1 \\ & (1- \mid\rho\mid^2)S_2S_2^{+}]\},\end{aligned} \qquad (3)$$

where $\nu_i = \sigma_i^2/\sigma_0^2$, $\mid f(\Delta u)\mid = \sin(N\Delta u/2)/(N sin(\Delta u/2))$, $\Delta u = (2\pi\delta/\lambda)(\sin\varphi_1 - \sin\varphi_2)$, $\alpha = 1 + N(\nu_1 + \nu_2) + N^2\nu_1\nu_2(1- \mid\rho\mid^2)(1- \mid f(\Delta u)\mid^2) + 2N\mid\rho\mid\bar{\nu}_1^{-}\bar{\nu}_2\mid f(\Delta u)\mid\cos\varphi$.

3. RESOLUTION OF SOURCE BEARING

When the sources are resolved, the beam energy evaluated at either target bearing must be larger than the beam energy evaluated between the target bearings. The criterion used for resolution of the sources is that the ratio of on-target to between-target beam energies exceeds a threshold value of 1 (Rayleigh resolution limit). For sources of identical intensity ($\nu_1 = \nu_2 = \nu$) situated near each other the threshold of resolution is defined by the condition

$$P(\varphi_i) \geq P((\varphi_1 + \varphi_2)/2), \; i = 1, 2. \qquad (4)$$

Then, making use of (3), from (4) we obtain

$$\begin{aligned}& 1 + 2N\nu + N^2\nu^2(1- \mid\rho\mid^2)(1- \mid f(\Delta u)\mid^2) \\ & + 2N\mid\rho\mid\nu\cos\varphi(\mid f(\Delta u)\mid - \\ & \mid f(\Delta u/2)\mid^2) + 2N^2\nu^2(1- \mid\rho\mid^2) \\ & \mid f(\Delta u)\mid\mid f(\Delta u/2)\mid^2 - 2N\nu(1 + N\nu(1- \mid\rho\mid^2)) \\ & \mid f(\Delta u/2)\mid^2 \geq 1 + N\nu(1- \mid f(\Delta u)\mid^2).\end{aligned} \qquad (5)$$

The complexity of expression (5) prevents a clear understanding of the effects of input signal-to-noise ratio (SNR), coherence, and bearing separation on resolving capability. To gain such an understanding, we will assume that the sources are localized in the main lobe and that the angular distance between them is small. Then, expanding the functions $\mid f(\Delta u)\mid$, $\mid f(\Delta u/2)\mid$ in an exponential series over Δu, we retain the first four terms of the expansion

$$\begin{aligned}& \mid f(\Delta u)\mid = 1 - \\ & (N\Delta u)^2/6 + (N\Delta u)^4/120 + (N\Delta u)^6/5040, \\ & \mid f(\Delta u)\mid^2 = 1 - \\ & (N\Delta u)^2/3 + 2(N\Delta u)^4/45 + 16(N\Delta u)^6/5040.\end{aligned}$$

As a result, for minimal Δu,from (5) we obtain

$$\Delta u = 8.71((1+ \mid \rho \mid \cos\varphi)/(N^5\nu(1- \mid \rho \mid^2)))^{1/4} \quad (6)$$

We can see that the increase in $\mid \rho \mid$ leads to a decrease in Δu; however, even in the case of strongly correlated sources resolution is possible if the input SNR is sufficiently large. For example, for $\mid \rho \mid= 0.99$ the minimum SNR is greater by $20dB$ than for uncorrelated sources for $\varphi = 0$. The relationship between $\Delta u$and φ indicates that the resolving power is determined by the location of the array. In particular, the array located at the maximum of the interference pattern ($\varphi = 0$) exhibits a resolving power lower than the array at the minimum of the interference pattern ($\varphi = \pi$). The maximum gain which can be achieved by changing the position of the adaptive array is determined by the quantity $(1+ \mid \rho \mid)/(1- \mid \rho \mid)$. For coherent signals, resolution independent of the input SNR becomes impossible, with the exception of the case in which $\varphi = \pi$. The effect of resolving the coherent sources for $\varphi = \pi$ when using adaptive beamforming was apparently first detected in [4] by computer investigations. However, it should be noted that estimates of the angular position of the sources are significantly shifted. Let's consider the case in which the angular distance Δu is on the order of the sbw. Then $\mid f(\Delta u) \mid\ll 1, \mid f(\Delta u/2) \mid\ll 1$. Condition (4) is satisfied for any $N\nu$. Proceeding from practical considerations, rewrite condition (4) in the form

$$P(\varphi_i) \geq \gamma P((\varphi_1 + \varphi_2)/2),$$

where $\gamma > 1$. The criterion used for detection of the sources, for example, is that $\gamma = 2$[2]. It then is not difficult to obtain the following condition

$$N\nu(1- \mid \rho \mid^2)^{1/2} \geq 1$$

We can see that the resolution of the coherent signals, even with a large angular distance between them, is impossible independent of the location of the array.

4. FAST FLUCTUATIONS OF ANTENNA APERTURE

In this section we conside a random lateral motion of the antenna in which the aperture line remains paralel to itself. The aditional phase increases caused by fluctuations of the coordinates will be different for each signal at each moment in time owing to the fact that the signals arrive from different directions. We shall assume that artificial Gaussian fluctuations with variance $\sigma_x^2$are introduced in the positions of the receivers along the aperture line. For each signal random Gaussian phase shifts equal to

$$\sigma_{1,2}^2 = (2\pi/\lambda)^2\sigma_x^2 \sin\varphi_{1,2}$$

One can see that introduction of these fluctuations in the position of the receiving elements decreases the coherent component (more accurately, it reduces the correlation coefficient) by a factor of $\exp((-1/2)\sigma^2)$, where

$$\sigma^2 = (2\pi/\lambda)^2\sigma_x^2(\sin\varphi_1 - \sin\varphi_2)$$

From (6) one can see that the gain in the resolving power can be write as

$$p = ((1+ \mid \rho \mid \cos\varphi)(1- \mid \rho \mid^2 \\ exp(-\sigma^2))/((1- \mid \rho \mid^2)(1+ \mid \rho \mid \\ \exp((-1/2)\sigma^2)\cos\varphi))^{1/4}$$

For coherent signals one can see that $p \Leftrightarrow \infty$.

5. FAST WAVEFRONTS FLUCTUATIONS

In this section we conside a simple uncorrelated signals case. We can write for the entries of the covariance matrix of each signals

$$\begin{aligned} \mathbf{R}_{lk} &= \sigma_0^2 + \sigma_1^2, \qquad l = k \\ &= \sigma_1^2 \exp(-j((2\pi\delta/\lambda)\sin\varphi_1(l-k)) \\ &\quad < \exp(j(\phi_l - \phi_k)) >, l \neq k \end{aligned}$$

where $\phi_l$is the phase error in l-element for first signal. For simplicity, we assume that these fluctuations are independent, stationary and with zero means, while their correlation times are much less than the weighting-coefficient adaptation times. We shall assume that these fluctuations have identical distributions in all channels. We can study the different kinds of distribution of these fluctuations. We limited the Gaussian distribution for each signal. Than we can write

$$\gamma = < \exp(j(\phi_{l,k})) > = \exp(-\epsilon/2),$$

where ϵ is the intensity of the phase perturbations. Finally, for covariance matrix of the single extertnal signal we can write

$$\begin{aligned} \mathbf{R} &= \sigma_0^2\mathbf{I} + \sigma_1^2 S_1 S_1^+ \gamma^2 + \\ & \quad \sigma_1^2(1-\gamma^2)\mathbf{I} \\ &= (\sigma_0^2 + \sigma_1^2(1-\gamma^2))\mathbf{I} \\ & \quad + \sigma_1^2 S_1 S_1^+ \gamma^2. \end{aligned} \tag{7}$$

For two external signals one can write

$$\begin{aligned} \mathbf{R} &= (\sigma_0^2 + (\sigma_1^2 + \sigma_2^2)(1-\gamma^2))\mathbf{I} \\ & \quad + \sigma_1^2 S_1 S_1^+ \gamma^2 + \sigma_2^2 S_2 S_2^+ \gamma^2. \end{aligned} \tag{8}$$

Comparison of (8) with (2) shows that these parameter fluctuations are equivalent to increasing the thermal noise power in proportion to the total power in the external source and to reduction in the external signal power as regards the effects on the stationary vector. Multiplicative parameter fluctuations in the channels and random independent detector movements tend to pump the external signals into the thermal noise. Hereat can shown that these fluctuations will not lead to decreases a correlation component between signals. From (6) we can write for gain in the resolving power

$$p = (\gamma^2/(1 + (1-\gamma^2)\nu))^{1/4}$$

From this expression one can see that in the presence of the powerful signals even relatively small phase errors can substantially reduce the superresolution properties of the adaptive antenna.

6. CONCLUSIONS

The capability of adaptive beamforming to estimate the bearings of two equal-energy correlated sources is examined. Analysis shows that it depends on the coherence magnitude and initial phase difference between signals. The effects of different kinds of perturbations are studied.

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