TIME-SPATIAL PROCESSING OF NEAR-FIELD MEASUREMENTS

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RÉSUMÉ

Le problème d'évalution de caractéristiques de sources acoustiques de bruit au moyen de mesures d'un champ voisin est considéré. Le calcul des algorithmes pour obtenir des estimations déplacées est démontré nécessire. Il y a deux algorithmes à proposer afin de traiter des données relatives aux mesures d'un champ voisin, à savoir: l'algorithm basé sur un comportement asymptomatique à hautes fréquences visant à résoudre une équation intégrale respective (CAHF-algorithme) et l'algorithme d'estimation au degré maximum de vérité avec régularisation (EDMR-algorithme). Certains résultats de simulation numérique sont présentés.

ABSTRACT

The problem of estimating the noise acoustical source characteristics from near-field measurements is considered. The necessity of algorithm design for obtaining biased estimations is shown. Two near-field data processing algorithms are proposed: the algorithm based on the high frequency approximation for solving appropriate integral equation — the HFA-algorithm and, the algorithm of the maximum likelihood estimation with regularization (the MLER-algorithm). Some results of numerical simulation are presented.

1. INTRODUCTION

At present the near-field methods (NFM) are broadly used for measuring direction pattern of microwave antenna. It is known that both the direction pattern and the amplitude and phase distributions on antenna can be determined by processing of near-field data measured in the region where the direction pattern is not formed. The main advantages of the NFM arising from nearness of radiating source to receiving system are possibility of decreasing a radiated power level and suppression of a reconstruction error component connected with propagation medium, reverberation effects etc. In acoustics the NFM can be generalized for possibility of measuring characteristics of extensive sound sources with complex unknown spectrum (for example, for diagnostics of noise radiation of cars, ships and so on). The final aims here are determination of angular distribution of radiation intensity in far field and reconstruction of elementary source distribution directly on radiator. For that we have been developing the NFM in following directions. Firstly processing algorithms have to be generalized for broadband signals having random nature. In this case the dependencies of acoustical field second moments on frequency and spatial co-ordinates must be estimated. Secondly special attention must be given to algorithm robustness against noise because in many real situations the measured acoustical signals are comparable with a noise background. Thirdly the radiator motion and the propagation conditions have to be taken into account.

To conclude, let us note that in low frequency range the most available measuring system is a linear antenna array. On the other hand, there are many types of acoustical radiators essentially oblong along one of the coordinate axes. The radiation of sources with such geometry can be described by elementary sources placed on a line segment. In the main we have been investigating the systems with a similar geometry.

1. GENERAL STRUCTURE OF PROCESSING ALGORITHMS. MODELS OF RECEIVED SIGNALS

When a noise stationary broad-band signal with unknown spectrum is radiated we estimate spectrum-angular distribution of radiation power in the far field $R(f,\theta) = \mathbb{E}\{|R^m(f,\theta)|^2\}$ and spectrum-correlation function $M(f,x_1,x_2) = \mathbb{E}\{m_f(x_1)m_f^*(x_2)\}$, where sign $*$ denotes complex conjugate, $\mathbb{E}\{.\}$ is the expectation operator, $R^m(f,\theta)$ is the momentary direction pattern for frequency f, function $m_f(x)$ describes the random elementary source distribution on radiator for frequency f. To this end we propose to divide the near-field data processing into four main stages:

1) current spectrum analysis (filtering signal of every receiving sensor by narrow bands);
2) spatial processing: transformation of array signal vector into momentary direction pattern vector or into momentary spatial distribution of elementary sources on radiator for every spectrum component and every time moments;
3) estimation of second order moments by temporal averaging with weight coefficients;
4) compensation of external noise influence.

The first stage of measured signal $\{s_n(t_i)\}$ processing is the calculation of current spectra. After the current spectrum calculation for every receiving sensor we have a set of vectors $\vec{p}_j = \{p_1^j, \ldots, p_{N_a}^j\}$ (where N_a is the number of receiving sensors, j is the number of time moment of current spectrum component) for every central frequency f_k. The vectors $\vec{p}_j$ can be considered as statistically independent for j.

After the current spectrum analysis the model of array signals is defined by:

$$\vec{p}_j = \vec{s}_j + \vec{n}_j = \sum_l \mathbf{G}_j{}^l \vec{m}_j^l + \vec{n}_j, \quad j = 0, \ldots, J-1, \tag{2.1}$$

where $\vec{n}_j$ is the vector of external noise on receiving sensors, the vectors $\vec{m}_j^l$ describe the momentary parameters of l-order multipole on radiator, $\mathbf{G}_j^l$ is the matrix of transformation coefficients from source to receiving sensors for frequency f.

We will use model of monopoles placed over $\lambda/2$ on radiator. The proposed model allows to synthesize a wide group of direction patterns. This choice is the result of compromise between the processing procedure complexity, its instability and the method error. Below index $l = 0$ will be omitted.

We assume that $\vec{m}_j$ and $\vec{n}_j$ are Gaussian with zero means and covariance matrixes $\mathbf{M}$ and $\mathbf{K}$ which do not depend on time:

$$\mathbb{E}\{\vec{m}_j \vec{m}_j^+\} = \mathbf{M}, \quad \mathbb{E}\{\vec{n}_j \vec{n}_j^+\} = \mathbf{K}, \tag{2.2}$$

where sign $+$ denotes conjugate transpose. If spectral power density of signal and noise are approximately constant in band Δf and temporal interval between counts equals $\frac{1}{\Delta f}$, then $\vec{s}_j$ and $\vec{n}_j$ are statistically independent. Then the covariance matrix of received signal $\vec{p}_j$ has the form:

$$\mathbf{P}_j = \mathbb{E}\{\vec{p}_j \vec{p}_j^+\} = \mathbf{G}_j \mathbf{M} \mathbf{G}_j^+ + \mathbf{K}. \tag{2.3}$$

The momentary direction pattern can be presented by the vector $\vec{R}_j^m$:

$$\vec{R}_j^m = \mathbf{U}\vec{m}_j, \quad \mathbf{U} = \frac{1}{r_s}\left\| e^{-2\pi i \frac{f}{c} x_l \sin\theta_k} \right\| \tag{2.4}$$

where r_s is some standard distance for normalization. We are interested in the estimation of angular distribution of radiation power (diagonal elements of matrix $\mathbf{R}$):

$$\mathbf{R} = \mathbb{E}\{\vec{R}_j^m \vec{R}_j^{m+}\} = \mathbf{U}\mathbf{M}\mathbf{U}^+ \tag{2.5}$$

or in estimation of the covariance matrix $\mathbf{M}$ of elementary sources on radiator. Thus the procedure of spatial processing consists in the estimation of momentary spatial distribution of elementary sources on radiator or in the estimation of momentary direction pattern from received signals:

$$\widehat{\vec{R}^m}_j = \mathbf{\Gamma}_j \vec{p}_j \tag{2.6}$$

The third and forth stages of proposed technique include time averaging with weight coefficients permitting to minimize the total estimation error, and the compensation of external noise influence:

$$R_{kk} = R(\theta_k) = \sum_j \gamma_{k,j} |\widehat{R^m}_{k,j}|^2 - R_{kk}^{comp}, \tag{2.7}$$

The matrix of spatial processing can be deduced from statistical and other criteria. The algorithm of high frequency approximation (the HFA-algorithm) and the algorithm of maximum likelihood estimation with regularization (the MLER-algorithm) are two different approaches to the determination of matrix $\mathbf{\Gamma}_j$.

3. THE HFA-ALGORITHM

For direct estimation of angular distribution of radiation power in the far field the HFA-algorithm can be designed:

$$\mathbf{\Gamma}_{kn}^{(HFA)} = \frac{d_a}{r_s}\sqrt{\frac{f}{c}\rho(x_n,\theta_k)}e^{-2\pi i \frac{f}{c} l(x_n,\theta_k)}, \quad (3.1)$$

$$\rho(x_n,\theta) = \frac{(\sin\theta \sin A z_n - \cos\theta \cos A\sqrt{z_n^2+\Delta h^2})^2}{\cos\theta\sqrt{z_n^2+\Delta h^2}}$$

$$l(x_n,\theta) = \sin\theta w_n + \cos\theta\sqrt{z_n^2+\Delta h^2}$$

$$z_n = x_n \sin A + (y_0 \cos A - x_0 \sin A)$$

$$w_n = x_n \sin A - (y_0 \sin A - x_0 \cos A),$$

where d_a is the distance between sensors, x_n is the co-ordinate of receiving sensor n, (x_0, y_0) is the arbitrary trajectory point co-ordinates in horizontal plane, A is the angle between source trajectory and array disposition line in horizontal plane, Δh is the difference of source and array disposition depths.

As follows from (3.1), $\mathbf{\Gamma}^{(HFA)}$ does not depend on the given location of radiator. This fact considerably simplifies the processing and so the main properties of obtained estimates can be studied on the basis of the HFA-algorithm. It can be shown that for every given location of radiator the angular distribution of radiation intensity is reconstructed with a small error only within a definite angular sector $[\theta_{1j}, \theta_{2j}]$. This sector will be called by a "trustworthy reconstruction sector" (TRS). From the results of numerical simulation we concluded that the reconstruction error within the TRS weakly depends on the concrete amplitude-phase distribution of elementary sources on radiator. Then for every motion trajectory the TRS can be constructed by numerical simulation for model source. The method of TRS construction consists in calculation of estimate $\widehat{R}_j^{mod}(\theta_k)$ and its comparison

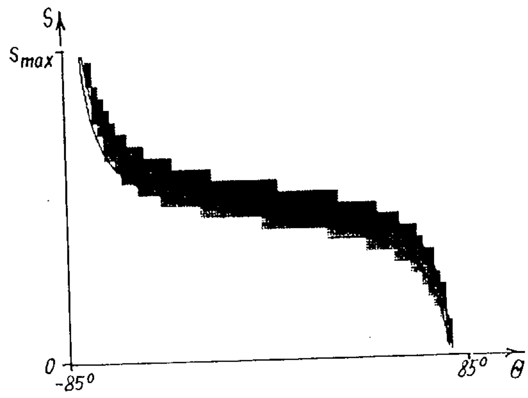

Figure 1: The transformation of trustworthy reconstruction sector on the plane $(\theta, S = vt)$ for a source motion parallel to the receiving array $(A = 0°)$

with exact model angular distribution $R_j^{mod}(\theta_k)$. The TRS transformation on the plane $(\theta, S = vt)$ is shown in Fig. 1 for parallel source motion relative to the receiving array $(A = 0°)$.

Taking into account this method of TRS determination the weight $\gamma_{j,k}$ in formula (2.8) is equal to 1 if the condition (3.2) is true and $\gamma_{j,k} = 0$ in the opposite case.

We found out that the robustness against noise deteriorates with an increasing receiving array length. This deterioration is connected with non-optimum of the HFA-algorithm because it does not take into account the finite source dimension and given source location on the trajectory. The other problem of using the HFA-algorithm is the difficulty of its generalization for cases when the propagation conditions considerably differ from free space. These circumstances restrict the possibilities of using the HFA-algorithm and one needs to design the optimum reconstruction algorithm. In the next section we propose the version of such algorithm design.

4. THE MLER-ALGORITHM

For model of received signal (2.1-2.3) the processing algorithm based on the known maximum likelihood principle can be designed. The likelihood function for statistically independent Guassian set $\vec{p}_j$ is determined as:

$$\ln \omega_p = -0.5 \sum_j [\vec{p}_j^{+} \mathbf{P}_j^{-1} \vec{p}_j + \ln det(\mathbf{P}_j)] + const \quad (4.1)$$

We will consider the reconstruction procedure only for matrix $\mathbf{R}$ because the one for matrix $\mathbf{M}$ is analogous.

We can assume that angles θ_k and the elementary source co-ordinates satisfy the relationship:

$$\frac{f}{c}(x_{l+1} - x_l)(\sin\theta_{k+1} - \sin\theta_k) = \frac{1}{L}, \qquad (4.2)$$

where L is the number of elementary sources. Then $\mathbf{U}$ is the unitary matrix: $\mathbf{U}^{-1} = \frac{r_s^2}{L}\mathbf{U}^+$ and we define $\mathbf{M} = \mathbf{U}^{-1}\mathbf{R}(\mathbf{U}^{-1})^+$. From (2.3) we get:

$$\mathbf{P}_j = \mathbf{H}_j\mathbf{R}\mathbf{H}_j^+ + \mathbf{K}, \qquad (4.3)$$

where $\mathbf{H}_j = \mathbf{G}_j\mathbf{U}^{-1} = \frac{r_s^2}{L}\mathbf{G}_j\mathbf{U}^+$. Differentiating (4.1) in $\mathbf{R}$ we obtain the non-linear equation for finding an asymptotically effective estimate $\mathbf{R}$:

$$\sum_j \mathbf{H}_j(\mathbf{P}_j^{-1}\mathbf{S}_j\mathbf{P}_j^{-1} - \mathbf{P}_j^{-1})\mathbf{H}_j^+ = 0, \qquad (4.4)$$

where $\mathbf{S}_j = \vec{p}_j\vec{p}_j^+$, $\mathbf{P}_j$ and the relationship between $\mathbf{P}_j$ and $\mathbf{R}$ are defined by (4.3). From equation (4.4) the linearized estimate has been obtained for an immovable source and generalized for a moving one. The processing procedure can be realized by the momentary direction pattern estimation and by the following trajectory averaging:

$$\widehat{\vec{R}^m}_j = \mathbf{B}_j^{-1}\mathbf{H}_j^+\vec{p}_j, \qquad (4.5)$$

$$\widehat{R}_{kk} = \sum_j \gamma_{k,j}{}^{(n)}|\widehat{\vec{R}^m}_{k,j}|^2 - R_{kk}^{comp}, \qquad (4.6)$$

where $\mathbf{H}_j = \mathbf{G}_j\mathbf{U}^{-1}$, $\mathbf{B}_j = \mathbf{H}_j^+\mathbf{H}_j =$ $= (\mathbf{U}^{-1})^+\mathbf{G}_j^+\mathbf{G}_j\mathbf{U}^{-1}$, $\gamma_{k,j}^{(n)}$ are the normalized weights, $\mathbf{R}^{comp} = \mathbf{B}^{-1}\mathbf{S}^{comp}\mathbf{B}^{-1}$, $\mathbf{S}^{comp} =$ $= \sigma_n^2\sum_j \mathbf{B}_j$ (for $\mathbf{K} = \sigma_n^2\mathbf{I}$).

The practical use of the algorithm (4.5)-(4.6) is connected with a possibility of $\mathbf{G}^+\mathbf{G}$ inversion. The fact of small eigenvalues in the matrix spectrum is very important for a spatial processing algorithm design. It means that modification of maximum likelihood algorithms is required to obtain biased estimates.

The modification method for obtaining biased estimates is based on model (2.1-2.3). We will assume that some determinate vector $\vec{m}$ (or $\vec{R}^m$) corresponds to every time moment j. Then the probability density ω_n of the vector $\vec{p}$ is determined by mean $\mathbf{G}\vec{m}$ (or $\mathbf{H}\vec{R}^m$) and covariance matrix $\mathbf{K}$. We will also assume that possible realizations generate the statistical ensemble with a priori probability density ω_{apr} characterized by mean $\vec{m}_{apr}$ and covariance matrix $\mathbf{M}_{apr}$. For deduction of spatial processing matrix we can

a) minimized the total estimation error $\mathbb{E}\{(\widehat{\vec{m}} - \vec{m})^+(\widehat{\vec{m}} - \vec{m})\}$ or $\mathbb{E}\{(\widehat{\vec{R}^m} - \vec{R}^m)^+(\widehat{\vec{R}^m} - \vec{R}^m)\}$, where $\widehat{\vec{m}}$, $\widehat{\vec{R}}$ are desired estimates;

b) maximized the generalized likelihood function $\omega_n\omega_{apr}$ in unknown parameters $\vec{m}$ (or $\vec{R}^m$).

If we do not have a priori information about distribution of elementary sources it is advisable to set $\mathbf{M}_{apr} = \frac{P_0}{L}\mathbf{I}$ from the information criterion. For this type of $\mathbf{M}_{apr}$ and $\mathbf{K} = \sigma_n^2\mathbf{I}$ the matrices of spatial processing are identical for cases (a) and (b):

$$\mathbf{\Gamma} = \mathbf{U}(\mathbf{G}^+\mathbf{G} + \varepsilon\mathbf{I})^{-1}\mathbf{G}^+,\ \varepsilon = \frac{L\sigma_n^2}{r_s^2 P_0} \qquad (4.7)$$

In the complete processing algorithm $\mathbf{G}$ and $\mathbf{\Gamma}$ must be replaced by current matrices. The parameter ε regulates the contributions of bias and dispersion to the total error. If ε decreases, then the bias decreases too, and the dispersion increases. The total error reaches the minimum value for the optimum ε. The optimum ε for one time realization is given by (4.7).

The final step of the MLER-algorithm design is the choice of weights. For this purpose we minimized the total estimation error for special type of $\mathbf{M} = \frac{P_0}{L}\mathbf{I}$ corresponding to the model of absolutely non-coherent source and for $\mathbf{K} = \sigma_n^2\mathbf{I}$. The weights $\gamma_{j,k}$ are found in explicit form for this case.

5. OUTLOOK

The results of an extensive simulation and the source characteristic reconstruction in natural experiments show that the estimates based on the above-mentioned algorithms have high accuracy. Furthermore, the MLER-algorithm allows to adapt the signal processing to complex source structure and to propagation medium by setting model Green's function $\mathbf{G}$ nearest the real one. It is believed that, besides being used for investigation of acoustic object noise radiation, the MLER-algorithm can be developed for broad sphere of applied problems.