ORTHOGONAL WAVELET PACKET TRANSMULTIPLEXING USING LATTICE STRUCTURES*

Andrzej DRYGAJLO **Benito CARNERO**

Signal Processing Laboratory, Swiss Federal Institute of Technology Lausanne, CH–Lausanne, Switzerland. email: carnero@elde.epfl.ch

RÉSUMÉ

Cet article décrit une nouvelle procédure générale pour l'élaboration de transmultiplexeurs orthogonaux à paquets d'ondelettes en utilisant des structures en treillis. Des schémas flexibles, basés sur des structures en arbre binaire et des structures de type polyphase, sont proposés pour la construction de transformations orthogonales recouvrantes permettant d'exécuter des opérations de synthèse et d'analyse sur des signaux. Un tel schéma permet d'effectuer une modulation orthogonale multirésolution en rapport avec le contenu temps-fréquence des signaux traités.

ABSTRACT

This paper describes a general procedure for the design of orthogonal wavelet packet transmultiplexers using lattice structures. Flexible schemes, based on binary tree structures and polyphase structures, are proposed to construct overlapped orthogonal transforms that serve to perform synthesis and analysis operations on signals. Such a scheme can achieve orthogonal multiresolution modulations in connection with the time-frequency contents of the processed signals.

1 Introduction

Multirate filter banks have been used in the realization of subband systems and transmultiplexers [1]. Nonuniform signal decomposition techniques by means of wavelet and wavelet packets have recently emerged as powerful tools for analysis, synthesis and processing of nonstationary signals such as speech and images [2] [3]. In a similar way, the importance of multicarrier modulation in communications-based scenarios has already been noticed in [4].

The main problems appearing in transmultiplexers are crosstalk cancellation between the different input signals (crosstalk-free transmultiplexers) and the elimination of amplitude and phase distortions (perfect-reconstruction transmultiplexers). The use of orthogonal synthesis/analysis structures in the construction of such systems guarantees perfect reconstruction and crosstalk cancellation in the modulation/demodulation process of the input signals. One major advantage of multi-channel orthogonal systems is also to ensure decorrelation among the different channel signals. The overlapped orthogonal transforms necessary for such applications can be apropriately derived from paraunitary filter banks [5].

Since wavelet packet transforms represent a general frame for multresolution time-frequency decomposition, they are well-suited for the elaboration of multiresolution multicarrier transmultiplexers.

2 Lattice structures

Lattice structures have been described in [5] as an elegant solution to the implementation of paraunitary filter banks, that possess the perfect-reconstruction property. Such structures are computationally efficient and preserve perfect-reconstruction after coefficient quantization. They are also well-suited to VLSI implementations, such as CORDIC processors [6].

A review and generalization of two-channel lattice structures has been presented in [7]. A polyphase lattice structure has been proposed as an equivalent alternative to the classical cascaded lattice (shown in Fig. 1). This structure, shown in Fig. 2 for filter lengths $L = 2, 4, 6$ and 8, possesses all the delay operators in the form of a delay chain. It is followed by an orthogonal overlapping block transform, exclusively composed of butterfly operators. We can assume, without loss of generality, that the lowest output branch, corresponding to $y_0(n)$, is related to a low-pass filtering operation and that the upper output branch of the rigthmost butterfly, corresponding to $y_1(n)$,

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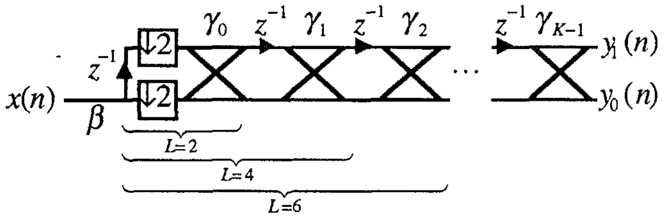

Figure 1: Two-channel cascaded analysis lattice structure: $\mathbf{H}(z)\downarrow_2$.

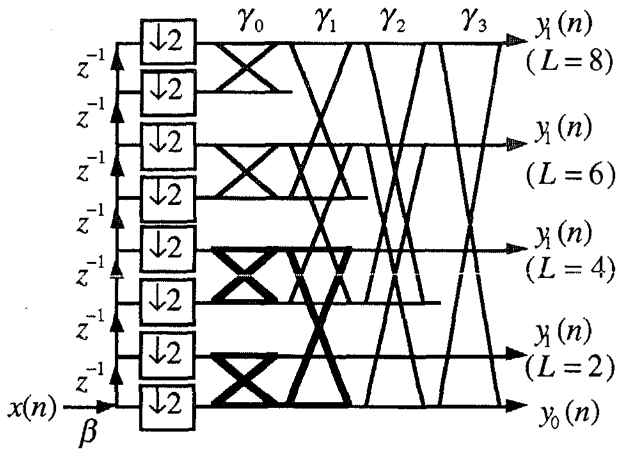

Figure 2: Two-channel polyphase analysis lattice structure. The structure necessary for filters of length 4 is highlighted.

is related to a high-pass filtering function. Depending on the number of butterfly stages ($K = 1, 2, 3, 4, \ldots$), we can build filters with different lengths ($L = 2K$). The factor β is a normalization factor [5]. The γ_i parameters ($i = 0, \ldots, K-1$) are the lattice rotation parameters [7]. They depend on the desirable filter characteristics and can be chosen in order to compute Daubechies filters [8], binomial filters [9], Malvar filters [10] or other paraunitary filter sets [11].

N-channel uniform tree structures and polyphase structures ($N > 2$) can be built by cascading the blocks of Fig. 1 or "growing" the polyphase lattice of Fig. 2 respectively, as explained in [12] [7]. Orthogonal wavelet packet transforms based on filters with different bandwiths and, therefore, different time-frequency resolutions, are chosen as substructures of these N-channel uniform systems. The examples of Fig. 4 and Fig. 5 illustrate how such transforms are built.

3 Complete multirate systems

We consider the general block scheme appearing in Fig. 3 for applications in communications-based subband systems. We have termed such a structure: *complete multirate system*. It is the cascade of analysis systems $\mathbf{A}_i(z)$, a transmultiplexer $\mathbf{C}(z)$–$\mathbf{D}(z)$ and

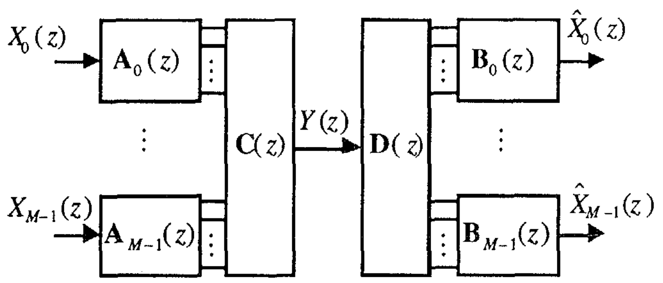

Figure 3: Complete Multirate System

synthesis systems $\mathbf{B}_i(z)$. The set of M input signals $\{X_0(z), \ldots, X_{M-1}(z)\}$ is decomposed by the nonuniform subband systems (analysis filter bank) $\mathbf{A}_i(z)$ to produce the input signals of the transmultiplexer. Such a decomposition is necessary in the case where subband processing is needed, as in coding, compression or scrambling. Once the subband signals have been processed, they are modulated by the synthesis system (synthesis filter bank) $\mathbf{C}(z)$ to produce the composite signal $Y(z)$. This signal is sent through a channel. A transmultiplexer should be adapted to the characteristics of the channel, which is generally not lossless, as well as to the time-spectral characteristics of the input signals, which are generally nonstationary. Finally, the different subband signals are retrieved by the analysis system $\mathbf{D}(z)$ and used to synthesize the M output signals $\{\hat{X}_0(z), \ldots, \hat{X}_{M-1}(z)\}$. If perfect reconstruction is achieved, the output signals should have the form $\hat{X}_i(z) = cz^{-p}X_i(z)$, where p is the total input/output reconstruction delay and c is an eventual scaling factor.

Since the transmultiplexer represents a modulation/demodulation operation and is the dual of an analysis/synthesis filter bank, we can use the lattice structures, developed for orthogonal decompositions, as basic building blocks for the design of multiplexing and demultiplexing systems. This means that nonuniform time-frequency transmultiplexing is achieved by employing multiresolution wavelet packet transforms.

4 Time-varying transmultiplexers

It is also interesting to consider transmultiplexing systems in a time-varying context. In such a case, the wavelet packet transforms performed on the signals are not fixed, but vary in time. This kind of processing may be needed because: the subband decomposition blocks $\mathbf{A}_i(z)$ are continuously modified to properly match the input signal characteristics; the transmultiplexer has to adapt to the changing characteristics of the channel. From the lattice point of view, two different time alterations can be considered: lattice parameters or lattice structure. The first deals with filter characteristics (phase and magnitude response) and, for example, their regularity properties [13].

The second deals with the time-frequency resolution of the system, since it implies modifications of the number of channels of the structure as well as the bandwidths. Time-varying signal processing must be performed according to a budget criterion (e.g. distortion, rate, entropy) in order to continuously find the best orthogonal wavelet packet basis in each time interval for signal synthesis and analysis [14] [15].

The overlapping polyphase structure is particularly interesting for the implementation of time-varying transmultiplexers. One major problem of time-varying tree-structured transmultiplexers is that the signal samples of pruned lattice branches are lost, when switching abruptly from one wavelet packet transform structure to another. This is a natural consequence of suppressing the delay blocks and the samples held by the delay blocks. This problem does not arise with the polyphase structure, since all samples are kept in the delay chain, which is never modified when different substructures are chosen. The idea is to adaptively reshape the lattice structure. In this case, it is necessary to know which substructure of the lattice should be activated in each moment of time to give the desired time frequency representation. Once time and frequency resolutions are decided, the corresponding substructure gives information on the samples that should be taken into account to provide the necessary calculations.

This approach allows us to find a transmultiplexing algorithm for fast implementations of time-varying wavelet packet bases, thereby maintaining perfect reconstruction throughout transitions. Such a procedure, for minimal-length one-level paraunitary filter bank realizations, was presented in [16].

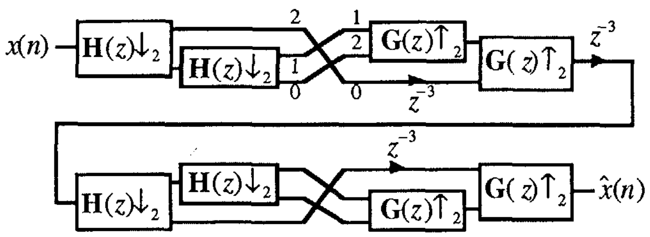

Figure 4: Tree-structured orthogonal transmultiplexing with scrambling. $\mathbf{H}(z)\downarrow_2$ are two-channel analysis filters banks and $\mathbf{G}(z)\uparrow_2$ are their synthesis counterparts.

5 Examples

As a potential application, besides the conventional multiplexing/demultiplexing, we consider a nonuniform multiresolution time-frequency scrambler that can be employed in secure communication contexts. It is implemented with tree structures, as shown in Fig. 4, and with polyphase structures, as shown in Fig. 5. To keep the illustration simple only one input signal $x(n)$ is processed. This signal is first decomposed by a wavelet packet filter bank. After this step, the different outputs are permuted, before being modulated by another wavelet packet transform, such that the frequency scrambling can be performed.

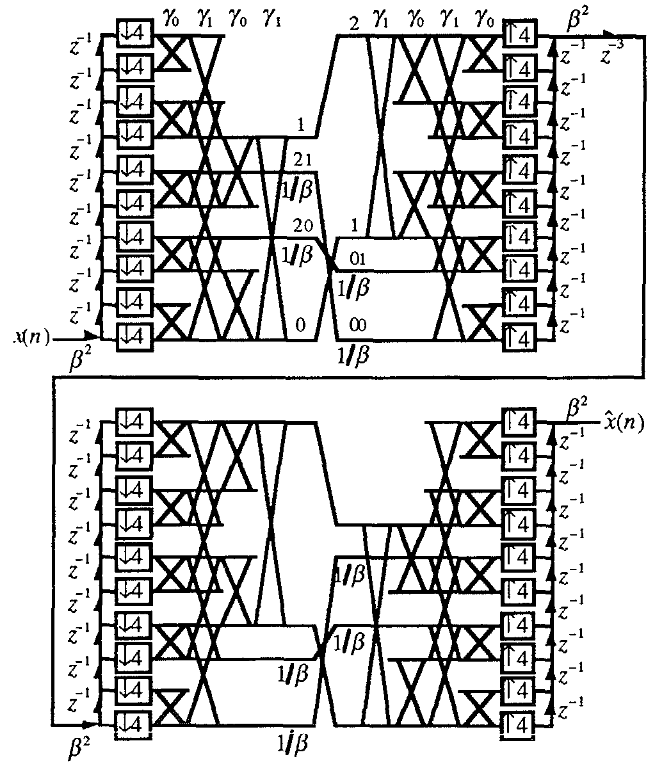

Figure 5: Polyphase orthogonal transmultiplexing with scrambling.

The permutation operation is depicted by the time-frequency tilings in Fig. 6 for the tree structure and those in Fig. 7 for the polyphase structure. The polyphase-based structure possesses one output for each tile of the time-frequency plane. The tree structure possesses only one output for each frequency band. This shows us clearly an advantage of polyphase structures over tree structures, since the spectral components with different time locations, in a given frequency band, are separately available for processing. We can see such spectral components by looking at channel 2 (e.g. 2_0 and 2_1) of the subband system or channnel 0 (e.g. 0_0 and 0_1) of the transmultiplexer in Fig. 6 or Fig. 7 (the first cifer is used for the band location and the second for the time location). These different spectral components within a band can be exploited to perform time scrambling, in addition to frequency scrambling, by changing the spectral component locations in time. Such an operation is depicted by the permutations $2_0 \rightarrow 0_1$ and $2_1 \rightarrow 0_0$.

Time scrambling can also be achieved with tree structures if the subband signals are delayed by different factors, however such a processing results in an increased input/output delay of the global system. The scrambled signal $Y(z)$ (or composite scrambled signal, in the case of several multiplexed inputs) is transmitted through the communication channel, which is considered to be lossless.

Finally, at the receiver end stage, $Y(z)$ is demultiplexed and descrambled in order to reconstruct $\hat{x}(n)$.

In these examples, the lattices correspond to length-4 filters. Such a choice leads to the delays appearing in the different branches of the structures in Fig. 4. For example, the lattice parameters can be set to $\gamma_i = \{1.7321, -0.2679\}$ for Daubechies filters, or to $\gamma_i = \{2.1999, -0.4142\}$ for Malvar filters, or to any other chosen value.

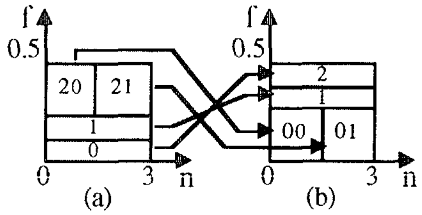

Figure 6: Scrambling and time-frequency representations achieved by the tree-structured system. (a) Subband system. (b) Transmultiplexer.

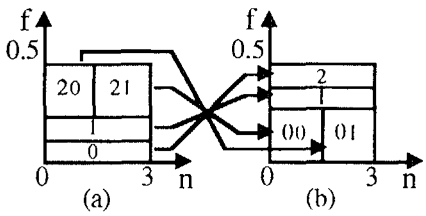

Figure 7: Scrambling and time–frequency representations achieved by the polyphase system. (a) Subband system. (b) Transmultiplexer.

We processed speech signals using the systems described in the above examples and were able to achieve relevant intelligibity degradation.

6 Conclusion

The design method proposed in this paper allows a high degree of flexibility in the construction of perfect-reconstruction wavelet packet transmultiplexers, well-suited for applications where non-uniform time-frequency processing is necessary. The suggested lattice-structured approach provides computationally efficient algorithms and ensures robustness under parameter quantization. Time-frequency resolution, subband frequency selectivity and input/output delay can be traded off for the order of the lattices. Tree structures and polyphase structures were used in the elaboration of multiresolution time-frequency transmultiplexing and scrambling examples. The advantage of using polyphase structures was justified in time-varying processing contexts. This approach is being examined currently.

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