QUATORZIEME COLLOQUE GRETSI - JUAN-LES-PINS - DU 13 AU 16 SEPTEMBRE 1993

Weighted Optimum Bit Allocation for Multiresolution Satellite Image Coding

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RÉSUMÉ

Dans cet article, nous proposons un algorithme d'allocation optimale des débits binaires qui prend en compte les caractéristiques de la chaîne de compression. Cet algorithme est basé sur la minimisation d'un critère non convexe. En effet, l'introduction de certaines contraintes, telle qu'une contrainte de positivité, peuvent rendre le critère non-convexe et empêcher l'algorithme d'allocation de converger.
Pour résoudre ce problème, nous proposons d'utiliser la méthode des Lagrangiens généralisés. Nous sommes alors amenés à introduire une **fonction de pénalité** dans le critère, qui assure l'existence d'un point col et la convergence de l'algorithme. Le problème de minimisation peut alors être résolu par l'utilisation d'une méthode classique de min max à l'aide du gradient conjugué.

ABSTRACT

This paper develops a new optimal bit allocation algorithm in order to have an efficient transmission of vector sources over a digital noiseless channel. This method takes into account the caracteristics of the compression scheme. This algorithm is based on the minimization of a non-convex criterion. In fact, the introduction of constraints, such as positivity constraint, could provide a non-convex criterion. Then, the optimization method may be caught in local minima.
To solve this problem, we propose to use the augmented Lagrangian method. Here, we introduce a **penalty function** which ensures the existence of a saddle-point and the convergence of the algorithm. Then, the minimization problem can be solved using a classical conjugate gradient method.

I. Introduction

Today, in digitized satellite image domain, the needs of high dimension images increase considerably. Then, the feasibility of the missions depends on the trade-off : transmitted image quality and transmission bit rates.
Thus, in most of the cases, an image coding process is performed on board, while the decoding algorithm is processed on earth. The coding algorithm must be adapted to the spatial mission. It depends on the input signal and the post-processing performed on the decoded image.
The input signal is described physically and statistically, and the knowledge of the optical instruments permits to construct a theoretical model of the imaging system or **Modulation Transfert Function** (MTF) of the imaging system. Furthermore, post-processings like **Digital Terrain Model** (DTM) are important in planetary missions, for the future satellites, or for piloting Automatic Planetary Rover (VAP).

In this paper, we propose a new optimal bit allocation algorithm associated to a data compression method, in order to reduce the bit rate for transmission or storage while maintaining an acceptable fidelity or image quality. A lot of research was recently done in this field [Brad 92, Rama 93]. However, the bit allocation method we propose here is based on the minimization of a non-convex criterion, and we give solutions to solve the convergence problems for this algorithm.

The proposed method can be combined with a multiresolution analysis structure like the wavelet transform, and can perform on every kind of signals (1-Dimensional, 2-Dimensional ...). The compression process is based on lattice vector quantization (for more information see [Anto 92, Barl 92]).

II. Quantization

II.1. Quantizer Distortion Approximation

For a large codebook (large number L of quantization symbols) and reproduction vector size n, Zador showed that the distortion $D(R)$ of a vector quantizer is given by [Zado 82] :

$$D(R) = A(n,2)2^{-2R}\left[\int_{\mathrm{IR}^n} f_X(x)^{n/(2+n)}dx\right]^{(2+n)/n} \tag{1}$$

where R is the bit rate in bits per sample (see paragraph II.2) and the values of $A(n,2)$ were tabulated by Conway and Sloane [Conw 85] for a uniform joint probability density function (pdf).

Making the asumption that X is an independent multidimensional variable, we can write

$$D(R) \leq A(n,2)2^{-2R}\left[\int_{\mathrm{IR}} p_Y(y)^{n/(2+n)}dy\right]^{(2+n)} \tag{2}$$

where $p_Y(y)$ is a monodimensional pdf.

II.2. Entropy

For the purposes of transmission or storage, a binary word c_i, of length b_i bits and called the index of the reproduction vector, is assigned to each output vector Y_i. Thus, vector quantization can also be seen as a combination of two functions : an encoder, which views the input vector X and generates the index of the reproduction vector specified by *Q(X)*, and a decoder, which uses this index to generate the reproduction vector Y_i. Let us define

$$b_i = -\log_2 p(Y_i)$$

where $p(Y_i)$ is the probability of selecting the reproduction vector Y_i during the encoding. Thus, the average binary word length, for a codebook Y is given by the formula

$$H(Y) = -\sum_{i=1}^{L} p(Y_i)\log_2 p(Y_i) \quad \text{bits/vector} \tag{3}$$

the so-called entropy measure of the codebook, which specifies the minimum bit rate necessary to achieve a distortion *D(R)* with the chosen quantizer. Hence, the practical average rate R in bits/sample, achievable with an entropy code (like Huffman or arithmetic code), is bounded by [Gers 92]

$$\frac{1}{n}H(Y) \le R \le \frac{1}{n}H(Y) + \frac{1}{n}$$

III. Bit Allocation Procedure

III.1. Problem

Let us define $D_T(R_T)$ the total distortion of a quantizer where R_T is the total bit rate we want to achieve. Then, in a multiresolution scheme, we have the two following equations :

$$\begin{cases} D_T(R_T) = \sum_i a_i\, D_i(R_i) \\ \text{and } R_T = \sum_i a_i\, R_i \end{cases}$$

where a_i are weighting resolution parameters which depend on the multiresolution scheme we use (see paragraph IV). These parameters exist, because the implementation of the wavelet transform is not isometric (see [Mall 89, Anto 92, Daub 88]). For example, in the classical dyadic case, $a_i = 1/2^{2i}$.

Each distortion $D_i(R_i)$ of each sub-image (sub-signal) is defined like the distortion *D(R)* given in paragraph II.1, formula (2). The R_i correspond to the practical average bit rates allocated to each sub-image (sub-signal).

III.2. Classical Method

A classical method of optimization consists in minimizing the following functionnal using Lagrangian multipliers

$$J_0(R_i, \lambda) = D_T^*(R_T) + \lambda\left[\sum_i a_i\, R_i - R_T\right]$$

where λ is a Lagrangian multiplier, and the weighted distortion is given by $D_T^*(R_T) = \sum_i a_i\, D_i(R_i)\, B_i$.

The assignment of the values B_i is based on the MTF of the imaging system and the post-processing applied on the quantized signal (see paragraph IV.2. for image coding application).
The solution obtained here is analytical and is given by [Anto 92]. Furthermore, a problem remains in the values of R_i which could be negative.

III.3. Optimization with a Positivity Constraint

In order to avoid negative values, we introduce in the criterion a positivity constraint (Cf. formula (4)) such that the bit allocation problem is formulated as

$$\begin{aligned} &\min_{R_i} D_T^*(R_T) \\ \underline{\text{subject to}} \quad & R_T = \sum_i a_i\, R_i \\ \underline{\text{and}} \quad & 0 \le R_i \le R_{\max} \end{aligned} \tag{4}$$

where R_{max} can be chosen as the maximum entropy of the sub-images or sub-signals. Then, the functionnal $J(R_i,\lambda)$ becomes

$$J(R_i,\lambda) = J_0(R_i,\lambda) + \mu\left[\sum_i \left(\frac{|R_i| - R_i}{2}\right)^2 + \sum_i \left(\frac{|R_i - R_{max}| + R_i - R_{max}}{2}\right)^2\right]$$

where $\sum_i \left(\frac{|R_i| - R_i}{2}\right)^2$ is the positivity constraint

and $\sum_i \left(\frac{|R_i - R_{max}| + R_i - R_{max}}{2}\right)^2$ is the constraint $R_i \le R_{max}$.

The choice for the value of μ, i.e. the positivity constraint in the functionnal, could provide a non-convex criterion. This is a typical ill-posed problem and the classical min max optimization methods may be caught in local minima.

III.4. A New Bit Allocation Scheme

However, a solution is given in [Rock 74, Flet 87 Chap.12] by the *augmented Lagrangian* method. Then, we must minimize the functionnal given formula (5).
Here, we introduce a penalty function which ensures the existence of a saddle-point and the convergence of the algorithm, even for large values of μ. In fact, the trade-off between μ and r permits to keep a convex criterion.
These values of μ and r are chosen by experiments according to the global bit rate R_T we want to obtain (see paragraph V).

$$J(R_i,\lambda) = J_0(R_i,\lambda) + r\left[\sum_i a_i R_i - R_T\right]^2$$
$$+\mu\left[\sum_i\left(\frac{|R_i| - R_i}{2}\right)^2 + \sum_i\left(\frac{|R_i - R_{max}| + R_i - R_{max}}{2}\right)^2\right] \qquad (5)$$

where $\left[\sum_i a_i R_i - R_T\right]^2$ is the penalty function.

Then, the minimization problem can be solved using a conjugate gradient method, and a solution is given by [Mino 83]

$$\underset{0 \le R_i \le R_{\max}}{\text{Min}} \quad \underset{\lambda \in \mathrm{IR}}{\text{Max}} \quad \left[J(R_i,\lambda)\right]$$

Note that other kind of augmented Lagrangian, based on the same idea (combination of the classical Lagrangian and penalty functions) were proposed in the litterature ([Naka 75] for example).

IV. Application to a Multiresolution Image Coding Scheme

IV.1. Principle

Multiresolution was recently introduced by [Meye 90, Mall 89] and extensively used by many researchers in image coding application [Mall 89, Anto 92]. The aim of the paper is not to present the theory of the wavelet transform, so the interested reader shall refer to [Meye 90, Daub 88].
Multiresolution exploits the eye's masking effects and therefore, enables us to refine the bit allocation according to the resolution level. Although a flat noise shape minimizes the MSE criterion, it is not generally optimal for subjective quality of images. To apply *noise shaping* across the sub-images, we define a total weighted MSE distortion $D_T^*(R_T)$. The weights B_i included in this distortion measure are chosen according to the post-processing as described in paragraph IV.2.

The orthonormal property of the wavelet decomposition ensures an additive contribution of the quantization error (MSE) across the scales and directions. The normalization we choose for the wavelet coefficients and the low frequency coefficients introduces an increase of the distortion in power of 4 in the dyadic case [Mall 89] and in power of 2 in the quincunx case [Barl 92] (this is due to a non-isometric implementation of the wavelet transform). For example, in the dyadic case the parameters a_i are equal to $1/2^{2i}$, so that the total distortion and the global bit rate can be written :

$$\begin{cases} D_T^*(R_T) = \frac{1}{2^{2I}} D_I^{dc}\left(R_I^{dc}\right) B_I^{dc} + \sum_{i=1}^{I} \frac{1}{2^{2i}} D_i(R_i)\, B_i \\ \text{and } R_T = \frac{1}{2^{2I}} R_I^{dc} + \sum_{i=1}^{I} \frac{1}{2^{2i}} R_i \end{cases}$$

where I stands for the lowest resolution (dc components) and $D_I^{dc}\left(R_I^{dc}\right)$ for the distortion of the lowest resolution with bit allocation R_I^{dc}.
For the computation of each theoretical distortion $D_i(R_i)$ we assume that the wavelet coefficient sub-images have Laplacian pdfs $p_X(x)$.

IV.2. Choice for the Weighting Factors B_i

In general, the problem of finding an optimal bit allocation is formulated so as to minimize a distortion measure between the original image and the quantized image. However, the choice of the distortion measure is often motivated by a numerical evaluation of the quality of the quantized image (like Peak Signal to Noise Ratio or Peak SNR) and not necessarily by its subjective quality for example.

In this section, we propose a strategy to take into account both the post-processing applied on the quantized image (visualization, correlation...) and the Modulation Transfert Function (MTF) of the imaging system (see Figure 2). These quantities are introduced in the weighting factors B_i (see Figure 1).
In fact, we assume that each MTF is of the type $H(f_x,f_y)$ in the Fourier domain. Thus, we can write the weight B_i in the Fourier domain as a product of the considered MTFs since it corresponds to a convolution product in the spatial domain :

$$B_i = \prod_m H_m\left(f_{x_i}, f_{y_i}\right)$$

For the X axis, the weighting function we use is plotted on Figure 1. It corresponds to the product, in the frequency domain, of the imaging system's MTF (low pass filter) and a correlation MTF (high pass filter). For the Y axis, the function is slightly the same. Here, the maximum size of the correlation window is taken equal to 9x9 pixels, and the frequency $f_e/2$ corresponds to 0.5 on figure 1.

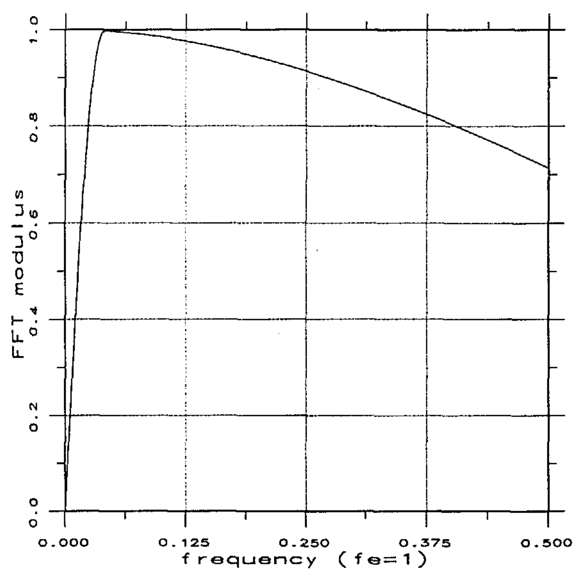

Figure 1 : Weighting function in the Fourier domain.

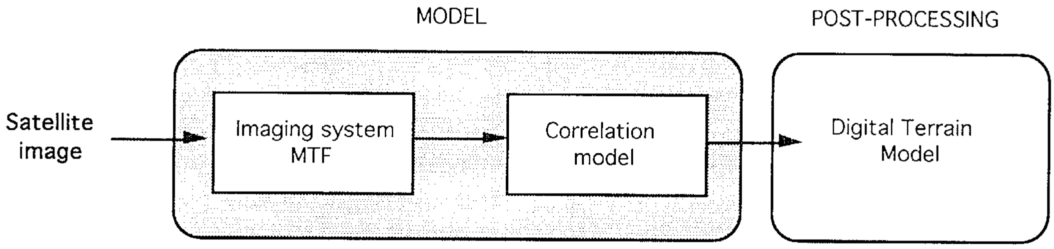

Figure 2 : Imaging system MTF, and post-processing model introduced in the optimization process.

V. Experimental results

The images we use for the experimental results are stereo pairs of 1024x1024 pixels satellite images, coded on 8 bits/pixel (8 bpp). These images, provided by the National Institute of Geography of France (IGN), are simulated images of Mars for the MARS 94 project. For this kind of images, the characteristics of the imaging system (MTF) and the post-processing (DTM) are perfectly determined.

A stereo pair is compressed with a compression ratio of about 20:1, which corresponds to 0.4 bpp. Here, two bit allocation schemes are used. The first corresponds to a bit allocation minimizing the usual MSE and the second, to a bit allocation using the weighting function given on Figure 1. In order to construct DTM, the disparity is computed on the coded/decoded stereo pair, using a correlation window of size 5x5 pixels. The results are compared to those obtained with the original stereo pair coded on 8 bpp. The results show that the total mean squared error of the disparity estimation decreases, when using a bit allocation procedure with weighting factors (see Table 1). This means that the quality of the DTM is better in the weighted case.

Disparity MSE	*X* axis	*Y* axis
B_i (of Fig.1)	30.86	29.46
B_i=1	33.71	30.96

Table 1 : MSE of the disparity estimation.

VI. Conclusion

The results we obtain are promising. In fact, we have construct an algorithm which permits to take into account both the MTF of the imaging system, the MTF of the correlation system, and all other MTF according to the post-processing we want to apply on the coded image.

We have introduced a positivity constraint and a maximal bound on the bit rate. Furthermore, using augmented Lagrangian operators, i.e. penalty function, this algorithm converge systematically towards an optimal solution. In fact, this method permits to solve the problem of non-convex criteria, encountered when introducing non linear constraints such as positivity constraint.

Acknowledgment

The authors whish to thank Dr. Laure Blanc-Féraud both from CNRS and I3S Laboratory for helpful discussions.

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