A MEAN CURVATURE MOTION MODEL APPLIED TO IMAGE PROCESSING

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RESUME

Dans ce travail, nous considérons un nouveau modèle pour le problème des contours actifs basé sur une équation aux dérivées partielles. Nous additionnons à l'équation de la courbure moyenne un terme qui représente une force constante dans la direction de la normale sur les ensembles de niveau et une fonction pour arrèter l'ensemble de niveau considéré sur le contour qu'on désire reproduire. Notre modèle est intrinsèque, stable (il satisfait le principe du maximum) et permet une analyse mathématique rigoureuse. De plus, nous pouvons déduire des algorithmes robustes et sans paramètres pour les applications. Nous présentons des expériences numériques et des applications sur images médicales.

ABSTRACT

In this work, we consider a new model for active contours based on a geometric partial differential equation. We add to mean curvature equation a term representing a constant force in the direction of the normal to the level sets and a function to stop the followed level set there we want to reproduced the desired contour. Our model is intrinsec, stable (satisfies the maximum principle) and permits a rigorous mathematical analysis. Moreover, we can design robust algorithms which can be engineed with no parameters in applications. Finally, we present numerical experiments applied to medical images.

1. INTRODUCTION AND MAIN RESULTS

One of the problems in image processing is the edge detection problem which is the problem of finding lines separating homogeneous regions. An edge must have two properties: to be smooth or piecewise smooth and the gradient of the image should be large along the edge. The classical snake method starts with an initial contour C_0 called "active contour" or "snake" near some contour γ_0 in the image and one looks for admissible deformations of C_0 which let it move towards the desired contour γ_0. These deformations are obtained by trying to minimize an energy functional designed in such a way that the set of local minima constitutes the searched image features. The original idea was due to Kass-Witkin-Terzopoulos [8], Blake-Zisserman [2] and further improvement of this model was successively done by Terzopoulos [13], Cohen [4], Cohen-Cohen [5] and many other contributions.

The energy functional always consists in the sum of two terms:

The internal energy E_{int} serves to impose a smoothness constraint. Representing the position of the snake parametrically by $v(s) = (x(s), y(s))$, $s \in [0,1]$, we can write the internal energy as:

$$E_{int} = \int_0^1 (\alpha|v'(t)|^2 + \beta|v''(t)|^2)dt \qquad (1)$$

According to the above mentioned authors, parameters $\alpha > 0$ and $\beta > 0$ impose the elasticity and rigidity coefficients of the curve.

The external energy E_{ext} depends on the features which are searched for in the image and is defined as:

$$E_{ext} = -\lambda \int_0^1 |\nabla (v(t))| dt \qquad (2)$$

where ∇u is the gradient of the image intensity and $\lambda > 0$.

Then, in the snake method, the moving curve tries to minimize the global energy $E_{int}+E_{ext}$. But the snake model has theoretical difficulties due to the fact that it is not an intrinsec model: the parametrization of the curves does not permit to get the geometrical regularity of the contours.

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In our work (see [3]), we propose a different model based on the mean curvature motion equation

$$\frac{\partial u}{\partial t} = |\nabla(u)| div(\frac{\nabla(u)}{|\nabla(u)|}) \quad (t,x) \in [0,\infty[x\Re^2 \quad (3)$$

which describes the motion of the level sets $\{x \in \Re^2 : u(x,t) = k\}$ of the function u, , $k \in \Re$, which evolve following the normal direction with speed depending on the mean curvature. In fact we add to (3) a term representing a constant force in the direction of the normal to the level sets and then multiply it by a function $g(x)$ to stop the followed level set there where we want to reproduce the desired contour. Thus our model is

$$\frac{\partial u}{\partial t} = g(x)|\nabla(u)|(div(\frac{\nabla(u)}{|\nabla(u)|})+\nu) \quad (t,x) \in [0,\infty[x\Re^2 \quad (4)$$

$u(0,x) = u_0(x) \quad x \in \Re^2$ where $g(x) = \frac{1}{1+(\nabla G_\sigma * g_0)^2}$, ν is a positive real constant, $G_\sigma * g_0$ is the convolution of the image g_0 where we are looking for the contour of an object O with the Gaussian $G_\sigma(x) = C\sigma^{1/2}\exp(\frac{-x^2}{4\sigma})$ and u_0 is the initial data which is taken as a smoothed version of the function $1-\chi_C$, where χ_C is the characteristic function of a set C containing O.

The geometrical interpretation of our model is the following:

1) The term $div(\frac{\nabla(u)}{|\nabla(u)|})$, which is teh curvature of the level set passing by x, ensures that the grey level at a point in ∂C increases proportionally to the algebraic curvature of ∂C at this point. This term is responsible for the regularizing effect of the model and plays the role of the energy term, E_{int}, in the snake model. The constant ν is a correction term chosen so that $div(\frac{\nabla(u)}{|\nabla(u)|}) + \nu$ remains always positive.

2) The term $|\nabla(u)|$ controls what happens at the interior and exterior of C.

3) The term $g(x)$ controls the speed at which ∂C moves. When ∂C is near the boundary of the object O, $|\nabla G_\sigma * g_0|$ is big and ∂C stops. We convolved the image g_0 to eliminate the effect of noise in our model. This coefficient is the point where the image comes in our model. Thus the energy criterium used to pusch the snake towards the desired contour in the snakes model has been replaced by a slowing down criterium represented by the coefficient $g(x)$ in our model.

We note that, as in the classical snakes, our model also gives an accurate localization of the edges and is able to extract smooth shapes. Moreover, it can provide several contours at no additional computational expense time and permits a theoretical analysis and proof of the correctness of the method. We remark that the evolution of the level set curves does not depend on its particular parametrization, while the snake model does.

Geometric equations like (4) have recently been studied in [6],[7], [12]. In particular, a related model of anisotropic diffusion with application to images was studied in [1]. All these equations satisfy a maximum principle and the basic mathematical tools employed are the theory of viscosity solutions for second order degenerate elliptic equations (see [6]).

If C denotes the level curve of u, an analogous model is given by

$$\frac{\partial C}{\partial t} = C_{ss} = curv(C(s))\vec{n}(s)$$

where s is the euclidian parameter and $curv(C(s))$ is the curvature of C at s. We remark that the theory of plane curve evolution has recently been introduced into computer vision by Kimia *et al* [9], [10] and [11] in order to regard the curve as the boundary of a planar domain which deforms in time. Our main theoretical result is:

Theorem. Let $u_0, v_0 \in C(\Re) \cap W^{1,\infty}(\Re)$. Then

1) The equation (4) admits a unique viscosity solution $u \in C([0,\infty[x\Re^2) \cap L^\infty(0,T;W^{1,\infty}(\Re^2))$ for all $T < \infty$. Moreover, it satisfies

$$inf_{\Re^2} u_0 \leq u(x,t) \leq sup_{\Re^2} u_0$$

2) Let $v \in C([0,\infty[x\Re^2)$ the viscosity solution of (4) with initial data v_0. Then for all $T \in [0,\infty[$ we have

$$sup_{0\leq t\leq T} \| u(t,x) - v(x,t) \|_{L^\infty(\Re^2)} \leq \| u_0(x) - v_0(x) \|_{L^\infty(\Re^2)}$$

where $W^{1,\infty}(\Re^2)$ denotes the space of bounded Lipschitz functions on $\Re^2$.

This result means in practice that the method will always work in completely reliable way : if $0 \leq u_0 \leq 255$, then $u(t)$ is defined in the same interval for every time.

2. NUMERICAL SCHEME AND EXPERIMENTAL RESULTS

In order to discretize the degenerate diffusion operator $|\nabla(u)|(div(\frac{\nabla(u)}{|\nabla(u)|}) + \nu)$, the numerical scheme is based on the one used by Alvarez-Lions-Morel in their work [1]. Thus, we only mention the general idea that was explained in more detail in their paper. First of all, the image is sampled on a grid. Of course, the discretizations of the differential operators at a point (i,j) of the grid, for obvious fastness and simplicity reasons must involve a few points around it. Typically, one would consider four other points for the discretization of the Laplacian, namely $(i\pm1,j)$ and $(i,j\pm1)$. Now, in our case, the differential operator can hardly be represented by two directions. Denote by $\xi = -x\sin\eta + y\cos\eta$, where

$(\cos\eta, \sin\eta) = \frac{\nabla u}{|\nabla u|}$, the coordinate in the diffusion direction (which is orthogonal to the gradient). The anisotropic term of the equation is therefore $\frac{\partial^2 u}{\partial \xi^2}$ and can easily be discretized only if in the direction $(-\sin\eta, \cos\eta)$ one can find points of the grid near (i, j). Anyway, we are led to a new formulation of the equation, which will take into account the discrete number of diffusion directions.

Let $0 \leq \eta_1 < \eta_2 < .... < \eta_n < \pi$ the n angles and $x_1, ..., x_n$ the coordinates defined by $x_j = -x\sin\eta_j + y\cos\eta_j$. In other terms, x_j is the coordinate orthogonal to the direction given by the angle η_j. We shall decompose the variable diffusion operator

$$\frac{\partial^2 u}{\partial \xi^2} = (\sin^2\eta)\frac{\partial u^2}{\partial x^2} - 2(\sin\eta\cos\eta)\frac{\partial^2 u}{\partial x \partial y} + (\cos^2\eta)\frac{\partial^2 u}{\partial y^2}$$

into a linear nonnegative combination of the fixed directional diffusion operators $\frac{\partial^2 u}{\partial x_j^2}$.

Consider the operator

$$Au = \sum_{j=1}^{n} f_j\left(\frac{\nabla u}{|\nabla u|}\right)\frac{\partial^2 u}{\partial x_j^2}$$

where the $f_j \geq 0$ are designed to be "active" only if $\frac{\nabla u}{|\nabla u|}$ is close to η_j. Then in order to discretize the degenerate diffusion operator $|\nabla(u)| div\left(\frac{\nabla(u)}{|\nabla(u)|}\right)$ we use the approximated operator defined before. The functions f_j define a "partition of unity". If the directions are given by $\eta_j = (n-1)\pi/2N$, $j = 1, ..., 2N$, then define an even smooth function f with support in $[-\pi/2N, \pi/2N]$ verifying $f(\pi/2N - \eta) + f(\eta) = 1$. Finally, the functions f_j are defined by $f_j(\eta) = f(\eta - \eta_j)$, where for simplicity the boundary points of the interval $[0, \pi]$ are defined.

After this aproximation, the algorithm follows and we obtain with this discretization a linear system in u^{k+1} which can be solved by any iterative method and for which sup$|u^{k+1}|$ $\leq$ sup$|u^k|$ (see [1] for more details).

The experimental results have been made on WorkStation SUN IPC, with a image processing environment called MEGAWAVE, whose author is Jacques Froment.

The parameters a priori necessary for the method are:

1) The value of the parameter ν, which represents the weight of the constant force in the direction of the normal to the level sets.

2) The choice of the level set which we want to follow through the motion. Depending on this choice, we find the edge of an object in different iterations and as a consequence of that, in different times.

3) The stopping time of the evolution. This parameter is automatic becausse we naturally impose a control coefficient constructed in the following form: If γ_t is the curve which represents the evolution of the initial curve given by the operator, we compute each iteration the function

$$E(\gamma_t) = \frac{1}{L(\gamma_t)} \int_{\gamma_t} |\nabla G_\sigma * g_0(x(s), y(s))| ds$$

where $L(\gamma_t)$ is the length of γ_t and we wait until $E(\gamma_t)$ decreases below a certain level, say $1/2E(\gamma_0)$, Then, we take as an optimal stopping time the time at which the function $E(\gamma_t)$ attains its maximum. This represents the time of best fitting between the envolving curve γ_t and the searched contour.

Some comments on the image

"l'echographie" is a medical image. In this image, one can see from left to right and from top to bottom the initial contour and the results of successive iterations.

3. REFERENCES

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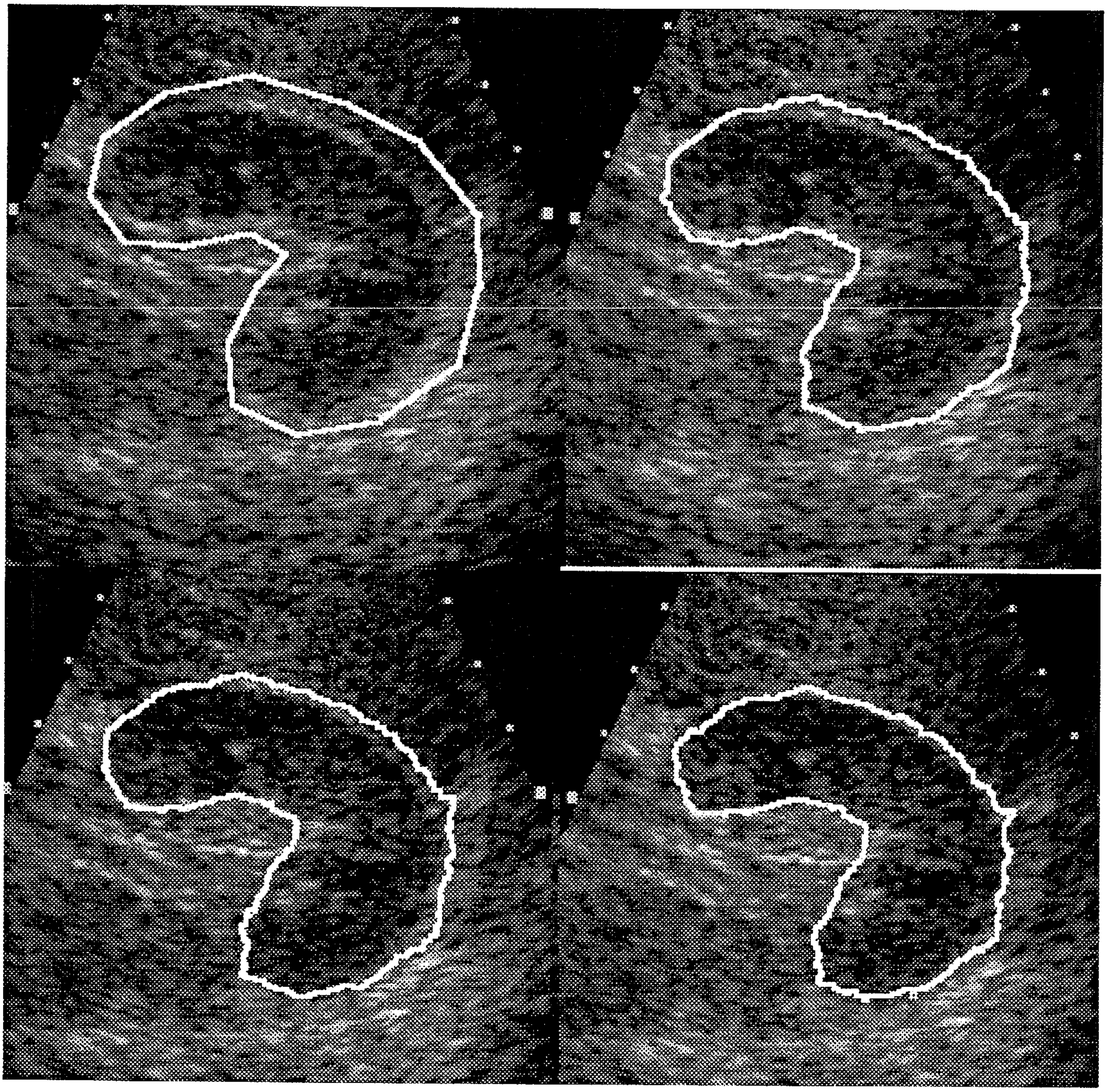

"L'echographie"