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On a Bound on Signal-to-Noise Ratio in Subband Coding of Gaussian Image Process

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ABSTRACT

The purpose of this paper is to analyze a bound on signal-to-noise ratio SNR in subband coding of Gaussian image process. For the proposed method optimization distortion-rate function as a fidelity measure is applied. The theoretical limit of a bound on SNR is obtained to be about 52 dB for a given Gaussian image power spectral density. The proposed method requires low computer cost because of its complexity compared to some other subband coding schemes.

RÉSUMÉ

Dans cet article on considère une limite du rapport signal à bruit dans le système pour le codage sous-bande d 'un procès d'image du type Gauss. Pour la méthode proposée on utilise l'optimisation de la fonction du distortion comme la mesure de la fidélité. La limitation théorique de la limite signal à bruit est à peu près 52 dB pour la densité spectrale de la puissance d'image Gauss. La méthode proposée donne des petits coûts pour le calculateur en comparaison avec quelques autres projets pour le codage sous-bande.

1 Introduction

One of the main goal in image subband coding is to remove the vast amount of redundancy which exists in the spatial domain as well as in temporal direction [1]. The general image data compression problem is achieving apriori tradeoff between an acceptable level of distortion for a given rate of information. This tradeoff is governed by the distortion-rate function. Since most image coding methods are statistical in nature, the distortion criteria usually used are also statistical-the more commonly used being a normalized mean squared error MSE or a signal-to-noise ratio SNR.

In the distortion-rate approach, the rate is constrained and the resulting average distortion is minimized [2]. It means that if the source is stationary, there exists a monotonically non-increasing distortion-rate function which provides a lower bound on the average distortion for a given rate, and hence an upper bound on the performance of practical image coders [3]. In the first part of this paper, after presenting unweighted distortion-rate function, we will analyze a bound on SNR. The second part of the paper represents some simulation results for the "Lena" test images.

2 Unweighted Distortion-rate Function

Let the statistics of the input Gaussian image process x(m,n) be represented by autocorrelation function

$$\Psi_{xx}(m,n) = \exp\left[-\alpha(m^2 + n^2)^{1/2}\right] \qquad (1)$$

The two-sided power spectral density is the Fourier transform of $\Psi_{xx}(m,n)$ and is given by

$$S_{xx}(f_m,f_n) = \frac{f_o}{2\pi}\,\frac{1}{(f_o^{\,2} + f_m^{\,2} + f_n^{\,2})^{3/2}} \qquad (2)$$

where $f_0 = \frac{\alpha}{2\pi}$. We define a unit area of the picture to be the area of a square whose side is the distance between points whose correlation is $\exp(-\alpha)$. Since the spectrum is symmetrical in f_m and f_n, the use of polar coordinates effectively eliminates one dimension. Transforming to polar coordinates, we obtain the distortion-rate function parametrically in the form

$$D(\theta) = 1 - 2\pi \int_{S(f_r)A(f_r)>\theta} f_r \, [S(f_r)\, A(f_r) - \theta] \, df_r \qquad (3)$$

where $A(f_r)$ represents the frequency weighting function, normalized so that the mean-square value of the weighted signal $A(f_r)$ $S(f_r)$ is unity. The parameter θ acts like a threshold. Namely, all frequency components with $S_{xx}(f_m, f_n) > \theta$ are coded and transmitted at the per pel $R(\theta)$, while the frequency components with $S_{xx}(f_m, f_n) < \theta$ are not transmitted. On the other hand

$$S(f_r) = \frac{f_o}{2\pi} \; \frac{1}{(f_o^{\,2} + f_r^{\,2})^{3/2}}$$

$$f_r = (f_m^{\,2} + f_n^{\,2})^{1/2} \qquad (4)$$

The spectrum $S(f_r)$ of (4) is monotonically decreasing in f_r and hence, for each θ there is a unique frequency which we call f_θ such that $\theta = S(f_\theta)$. Then the distortion-rate function in parameter form becomes

$$D(f_\theta) = \pi f_\theta^{\,2} S(f_\theta) + 2\pi \int_{f_\theta}^{\omega} f_r \, S(f_r) \, df_r \qquad (5)$$

From equations (4) and (5), we obtain

$$D(f_\theta) = \frac{f_\theta^{\,2} f_0}{2\,(f_0^{\,2} + f_\theta^{\,2})^{3/2}} + \frac{f_0}{(f_0^{\,2} + f_\theta^{\,2})^{1/2}} \qquad (6)$$

The quantity f_θ represents so called the "throw-away" frequency. Namely, the signal is bandlimited to f_θ, while the signal energy at radial frequencies greater than f_θ is not transmitted. The first term in $D(f_\theta)$ of (5) and (6) is the in-band quantizing noise. On the other hand, the second term is the bandlimiting distortion.

3 On a Bound on SNR

The proposed SBC system with two bands is shown in Fig.1. A source signal $x(m,n)$ is passed through the two analysis filters $G_a(f_m, f_n)$ and $H_a(f_m, f_n)$ which are high and low-pass, respectively. The filtered signals are decimated by a factor of 2x2. After coding, transmission and decoding, the filtering process is carried out in reverse. Since the input signal is stationary and Gaussian, the two subbands signals are also stationary and Gaussian with spectral densities $S_{hh}(f_m, f_n)$ and $S_{ll}(f_m, f_n)$. The respective distortion functions are [4]

$$D_{ll}(R_{ll}) = \sigma_{ll}^{\,2}\, \gamma_{ll}^{\,2}\, 2^{-2Rll}$$

$$D_{hh}(R_{hh}) = \sigma_{hh}^{\,2}\, \gamma_{hh}^{\,2}\, 2^{-2Rhh}$$

Here, $\sigma_{ll}^{\,2}$ and $\sigma_{hh}^{\,2}$ are the variances, while $\gamma_{ll}^{\,2}$ and $\gamma_{hh}^{\,2}$ denote the spectral flatness measures, for the low and high band, respectively.

The optimum overall coding performance is found by choosing the bit rates R_{ll}, R_{hh} low and high band, respectively, to minimize

$$D_{SBC}(R) = D_{ll}(R_{ll}) + D_{hh}(R_{hh})$$

subject to the constraints

$$R_{ll} + R_{hh} = 2R \; ; \qquad R_{ll},\, R_{hh} \geq 0$$

The resulting distortion is given by the well known formula

$$D_{SBC}(R) = 2 \sqrt{\sigma_{ll}^{\,2}\, \gamma_{ll}^{\,2}\, \sigma_{hh}^{\,2}\, \gamma_{hh}^{\,2}} \;\; 2^{-2R} \qquad (7)$$

It should be noticed that in the practical use $D(f_\theta) \leq D_{SBC}(R)$, when the equality holds in the ideal case. It means that the subband coding must be suboptimum in a rate-distortion sense compared to encoding the source directly. Bearing in mind this fact, we can write

$$\frac{f_\theta^{\,2} f_0}{2\,(f_0^{\,2} + f_\theta^{\,2})^{3/2}} + \frac{f_0}{(f_0^{\,2} + f_\theta^{\,2})^{1/2}} \leq$$

$$\leq 2 \sqrt{\sigma_{ll}^{\,2}\, \gamma_{ll}^{\,2}\, \sigma_{hh}^{\,2}\, \gamma_{hh}^{\,2}} \;\; 2^{-2R}$$

where f_θ and f_0 are previously defined, while the value $d = 0{,}05$ was chosen because correlation between picture elements-pels of about exp(-0,05) ≅ 0,95 is typical for pictures of moderate to low detail. In the relation (7), subroot values are

$$\sigma_{11}^{2}\,\gamma_{11}^{2} = \exp\left[\int_{-1/2}^{1/2}\int_{-1/2}^{1/2} \ln S_{11}(f_m,f_n)\, df_m\, df_n\right]$$

$$\sigma_{hh}^{2}\,\gamma_{hh}^{2} = \exp\left[\int_{-1/2}^{1/2}\int_{-1/2}^{1/2} \ln S_{hh}(f_m,f_n)\, df_m\, df_n\right] \qquad (8)$$

where the expressions on the right side of the above equations (8) denote the entropy power of the observed subband image signals.

If ideal "brickwall" filters are used the resulting distortion equals the source distortion, i.e. $D(f_\theta) = D_{SBC}(R)$, Using equations (4) and (5), we obtained from equation (6) that $D(f_\theta) = 0,39$. In this case, the peak-to-peak signal-to-noise ratio becomes

$$PPSNR = 10 \log \frac{255^2}{D_{SBC}(R)} = 52,22 \text{ dB}$$

The 52,22 dB is a bound on signal-to-noise ratio in subband coding for Gaussian image process.

4 Simulation Results

The experimental analysis was performed for the subband coding scheme as previously described. The spliting scheme was used with the QMF filters with 16 taps. A normalized transitional bandwith was 0,14 radian. The weighting factor is 1, while the reconstruction error was 0,0008 dB. The magnitude responses of the proposed filters expressed in dB as a function of ω are shown in Fig.2. The stopband attenuation varies from 60 dB(first peak) to 90 dB(last peak). The filtering was performed in the Discrete Fourier Transform DFT frequency domain with 256x256 image size.

In our experiment we observed that for the low band the pel to pel correlation is very high. Therefore, the differential pulse code modulation DPCM method is only considered to encode the low band. Our use of DPCM is motivated by the increased efficiency of the predictive coder. Namely, for DPCM the reconstruction mean-square error MSE is equal to the quantizer MSE. On the other hand, the high band signal is coded by pulse code modulation PCM. We have observed that DPCM quantizer is not suitable in a subjective sense for coding the high frequency band signal. This is mainly due to the existence of camera noise which manifests itself as a low level signal within this band and would result in a fine quantization of the noise.

Fig.3 shows the coding results for the "Lena" image using a SBC approach with two subbands. Fig.3.a represents the original image, while Fig.3.b is a subimage PCM coded in the high band with a 3-level uniformed quantizer, i.e. with 1,58 bit per pel-bpp. After the DPCM coding, in the low band, we obtained the subimage shown in Fig.3.c. The experiment was carried out with 1,58 bpp, too. The fourth image, i.e. Fig.3.d is obtained after the process of the interpolation as well as reconstruction filtering in a high band. The same process in a low band gives the "Lena" image shown in Fig.3.e. Finally, the reconstructed "Lena" image of a good quality is performed in Fig.3.f by summing the subimages represented in Fig.3.d and Fig.3.e.

5 Conclusion

The presented simulation results demonstrate that good quality monochrome pictures can be obtained at a bit rate of about 1,5 bits per pel for a two-bands SBC scheme. On the other hand, the theoretical limit of a bound on SNR is obtained to be about 52 dB for a given Gaussian image power spectral density. It should be added that the proposed method requires lower computer costs because of its simplicity, compared to some others SBC coding schemes.

Subband coding should be considered as one of the most attractive Gaussian image coding process. Suggestions for further study include the optimization in the sense of balance between three parameters, i.e.: (a) compression ratio, (b) reproduced image quality as well as (c) economical effects. Theoretic bounds of the SNR for a suitable separable process approximating real images would be of interest, too.

References

[1] J.W.Woods and S.D.O'Neil, "Subband coding of images", IEEE Trans.Acoust.Speech, Signal Processing,

vol.ASSP-34, pp 1278-1288, Oct.1986
[2] T.Berger, Rate Distortion Theory, Englewood Cliffs, NJ:Prentice-Hall, 1971
[3] Z.Bojković:"Some Results in Image Subband Coding", presented at The University of Texas at Arlington, Department of Electrical Engineering Seminar, April 1993
[4] N.S.Jayant and P.Noll, Digital Coding of Waveforms, Englewood Cliffs, NJ: Prentice-Hall, 1984

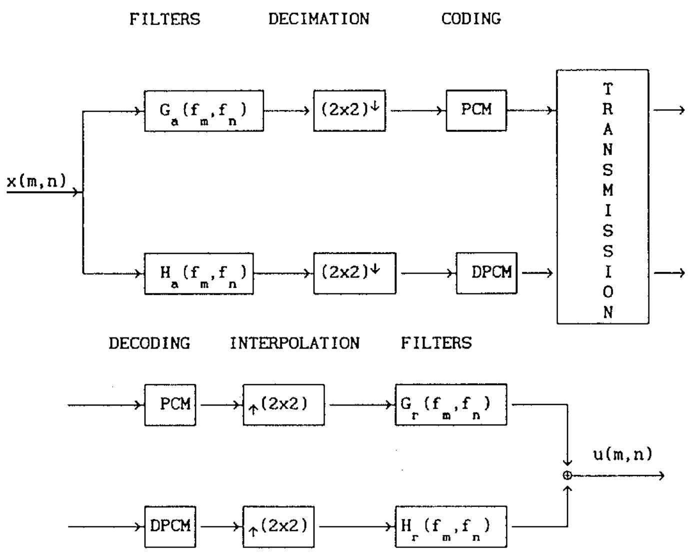


Fig.1. Subband coding system with two bands

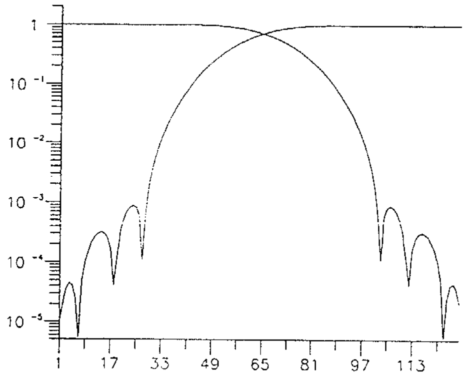
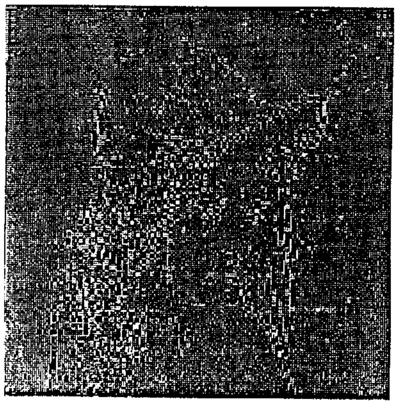


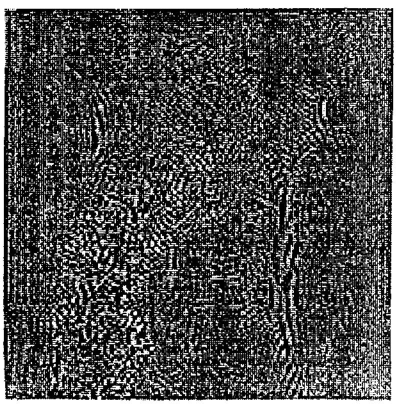
Fig.2. Magnitude response of the QMF filters

a)



b)

c)



d)

e)

f)

Fig.3. SBC results for "Lena" image: a)original image, b)PCM coded at 1,58 bpp, c)DPCM coded at 1,58 bpp, d)reconstructed high band, e)reconstructed low band, f)reconstructed image