



Image Compression by local Symmetries extraction, and Entropy Coding by Vector Quantization and Arithmetic Coding

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Résumé

Dans cet article nous présentons une nouvelle méthode de compression d'images. Celle-ci se base sur l'extraction de paramètres de symétrie présents dans les scènes naturelles. Après diminution de la redondance par exploitation de la symétrie, l'information de luminance est approximée par un polynôme. Les axes de symétrie sont codés de façon entropique. L'information de luminance est comprimée par une méthode combinant la quantification vectorielle et le codage arithmétique. Après une description du système de compression, des résultats de simulation sur des images statiques sont présentés.

1 Introduction

Recent efforts towards reduction of redundancy in images showed the relevance of modeling them as a combination of nonstationary visual primitives [1]. This led to significant improvement in compression. As an example, segmentation-based or contour-texture-based coding demonstrated superior performance with respect to classical waveform coding whose fundamental assumption remains the stationarity of the source. If contours or areas with uniform textural characteristics seemed adequate to be chosen as primitives, more elaborate patterns or properties that can be found in natural scenes may be considered for better performance as well.

In a recent work [2], we proposed a novel technique which suggested the use of symmetry. The introduction of this geometric property is dictated by the fact that natural objects often give rise to the human sensation of symmetry. This sense of symmetry is so strong that most man-made objects are symmetric, and this concept is more general than the strict mathematical notion [3]. For instance, a picture of a human face is considered highly symmetric, although it is not symmetric in the mathematical sense. Furthermore, we

Abstract

In this paper we propose a new technique to compress images. It is based on the extraction of symmetry features which are present in natural scenes. After the redundancy reduction by symmetry extraction, a polynomial approximation on the luminance information is performed. The symmetry axes are entropy coded, while the luminance information of the polynomial approximation is coded by an entropy coding scheme which is a combination of vector quantization and arithmetic coding. Description of the coding system and simulation results, on natural still images, are presented.

present a lossless entropy coding scheme which combines vector quantization (VQ) and arithmetic coding [4].

This constitutes an improvement with regard to [2], both in terms of quality and compression.

2 System Description

In the following, the scheme of the coder is described (see Fig. 1). In order to adapt to local symmetries in the image data, the system is implemented in a block-based fashion. For each block the axis of symmetry is found on the basis of the luminance function $I_i(x, y)$. The technique is extended to non-symmetric blocks by the introduction of a Coefficient of Symmetry β . Then the linear prediction of one part of the image is built from pixels symmetrically chosen on the other side of the axis of symmetry. The information relative to each block are the location and orientation of the axis of symmetry, together with the luminance on one side of it. The luminance information is compacted by a 2D polynomial approximation. Axis position and polynomial coefficients are entropy coded.

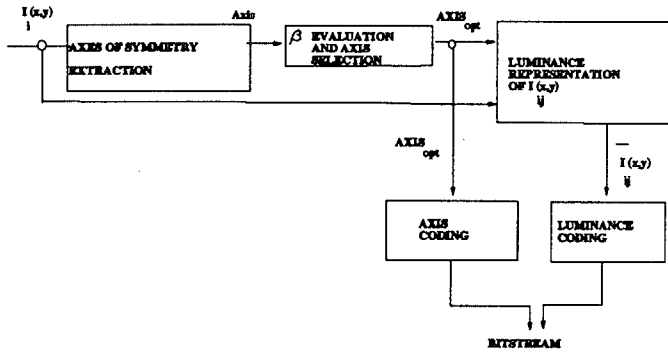


Figure 1: Encoder block diagram.

3 Image Symmetry Extraction and Representation

Several techniques for extracting axes of symmetry of objects have been proposed, mainly for pattern recognition applications. In [2], we proposed a simple and efficient technique, which identifies axes of symmetry to the Principal Axes of Inertia (PAI) of the rigid body defined by the image plane and the luminance function [5]. In fact, it can be demonstrated that, if an object has an axis of symmetry, this axis is also a PAI. Conversely, if an object does not have a real symmetry, the PAI partitions the body "optimally" into two quasi-symmetric parts [5]. The technique which finds the PAIs of the object defined by the image plane and the luminance function is followed by the definition of a Coefficient of Symmetry, which allows to measure the degree of symmetry associated to each PAI. Let F be an image with an axis of symmetry and $g(x, y)$ its associated luminance function. If $P(x, y)$ and $\bar{P}(\bar{x}, \bar{y})$ are two points symmetric with respect to a certain axis d (see Fig. 2), we define the quantity $\beta \geq 0$ as follows:

$$\beta = 1 - \frac{\frac{1}{2} \iint [g(x, y) - g(\bar{x}, \bar{y})]^2 dx dy}{\iint g^2(x, y) dx dy}, \quad (1)$$

$$\text{with } 0 \leq \beta \leq 1. \quad (2)$$

If $\beta = 0$ the axis is outside F ; if $\beta = 1$ F is symmetric; β is called Coefficient of Symmetry of an axis.

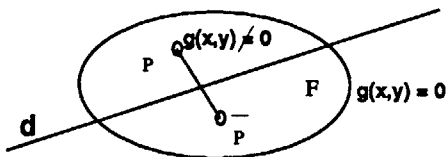


Figure 2:

By retaining the PAI associated to the highest symmetry measure, a given image will be separated into the two most quasi-symmetric parts, according to the Coefficient of Symmetry measure. Once a symmetry axis has been found, we propose to build the linear

prediction of one part of the image from the opposite part measured symmetrically from the chosen PAI. In other words, each pixel belonging to one part of the image is linearly predicted from pixels symmetrically chosen on the other side of the selected PAI. In the following, results are presented using a linear prediction of order zero. That is, a pixel is obtained from the one which is symmetrically located with respect to the chosen PAI. The method which has been previously described is implemented in a block-based fashion in order to adapt to local symmetries of the image data. For each block the system finds the two PAIs on the basis of the luminance function. Then the Coefficient of Symmetry associated to each PAI is evaluated; according to this measure, the PAI corresponding to the highest value is selected. The luminance representation of each block is obtained by coding separately the PAI location and orientation and the luminance on one side of it. According to visual evaluations performed on different test images, the prediction error is not transmitted. As far as luminance representation is concerned, the problem is to find a representation which approximates the given gray values and describes them in a compact form allowing successive data compression. One possible choice is to use the *Polynomial Basis Functions*:

$$\varphi_i(x, y) = x^{k(i)} y^{l(i)}, \quad \text{for } k(i) + l(i) \leq n. \quad (3)$$

In the following, results of the technique are given when using a 2D second order polynomial function. In the following sections, a coding strategy specifying the selected PAIs and the luminance on one side of them is defined. The information to be coded are the 6 coefficients per partition of the polynomial approximation, together with two parameters specifying the PAIs location and orientation. The coefficients of the polynomial approximation have to be represented using a high number of bits in order to reach the necessary precision. This representation is expensive in terms of bit cost; therefore an alternative representation of the information is proposed. We define the positions of 6 pixels, fixed for all the blocks. The quantized values of these 6 pixels allow the unique reconstruction of the approximating polynome. This is accomplished by a simple 6×6 matrix inversion. The polynomial approximation of the luminance is represented by a set of 6 pixel values which have to be opportunely coded.

4 Representation and Coding of the PAIs

A set of image points (x, y) , which lie on a straight line, can be defined by a relation f , such that

$$f[(m, c), (x, y)] = y - mx - c = 0, \quad (4)$$



where m and c are two parameters, the slope and intercept, which describe the straight line.

Straight lines might better be parameterized by the length ρ , and orientation θ of the normal vector to the line from the image origin, where

$$\rho = x \cos \theta + y \sin \theta . \quad (5)$$

This representation has advantages over the (m, c) parameterization which has a singularity for lines with large slopes, that is, for $m \rightarrow \infty$. It is clear that, for a block of size $x_{max} \times y_{max}$, the values of ρ and θ are bounded by

$$-R < \rho < R \text{ and } 0 < \theta < \pi , \text{ where} \quad (6)$$

$$R = \sqrt{x_{max}^2 + y_{max}^2} . \quad (7)$$

In this case, a uniform quantization, confined to the region $-\frac{\pi}{2} \leq \theta \leq \frac{\pi}{2}$, $-\frac{R}{2} \leq \rho \leq \frac{R}{2}$, of the two line parameters (ρ, θ) has been used.

5 Luminance coding using VQ and arithmetic coding

The final part of the system consists of coding the 6 pixel values spread over each 8×8 block with the lowest bit cost to reach the highest possible compression ratio. The coding has to be lossless because any error or distortion introduced in the luminance pixel values affects the polynomial reconstruction, which is highly sensitive.

Transform based methods are efficient in compression only when they can be used in a lossy way. Methods based on a linear prediction like DPCM or ADPCM are usually used for compression when linear statistical dependencies among the samples are present. In our case, where the pixels are not adjacent one to each other but are distributed on a block, linear statistical dependencies (correlation) is reduced. Therefore the efficiency of linear prediction schemes is decreased.

We propose a scheme (see encoder block diagrams in Fig. 3) that considers not only linear but also non linear dependencies by means of VQ. VQ is an intrinsic lossy operation, hence it requires a correction of the error e_{VQ} introduced by the VQ stage. This error can be considered as being a memoryless signal source. It is entropy coded using arithmetic coding. The global bit rate is given by the sum of R_{VQ} and the arithmetic coder rate R_{AC} . We will show that $R_{AC} + R_{VQ}$ rate is lower than the entropy of the original luminance signal values. This means that our scheme succeeds in exploiting statistical dependencies among the data (VQ). Without an explicit modeling of the statistical dependencies, VQ is able to consider them implicitly in the training of the codebook table. This constitutes an advantage over DPCM and ADPCM schemes, which

require the explicit construction of a model. This operation is difficult when the dependencies structure is unknown.

Arithmetic coding can reach the entropy of an ergodic stationary discrete source without the power-of-two constraint on the discrete probability density function (dpdf). On the contrary, Huffman coding does not have this property. Arithmetic coding is also characterized by a low computational complexity.

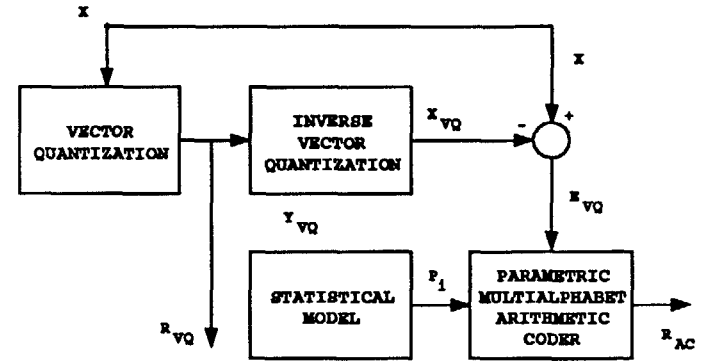


Figure 3: Block diagram for luminance entropy coding.

The 6 pixels from each block are combined in a vector x and are vector quantized, resulting in a bit rate R_{VQ} . The LBG algorithm has been selected to train the codebook tables. The training set is made of data coming from 10 different images. A full search algorithm is used for the VQ encoding.

The error introduced by VQ:

$$e_{VQ} = x - x_{VQ} \quad (8)$$

is computed and is entropy coded by an arithmetic coder. The statistical model $\{Q_i\}$ is a zero-mean discrete probability density function (dpdf).

The arithmetic coder encodes each symbol with a number of bits equal to the self information of each symbol according to the model:

$$l_i = -\log_2 Q_i . \quad (9)$$

The task is to study the statistics of VQ error e_{VQ} in order to model a dpdf $\{Q_i\}$ for the arithmetic coder. It is well known that the entropy of the source can be reached only when the model dpdf $\{Q_i\}$ is equal to the pdf of the source $\{P_i\}$. The choice of the model $\{Q_i\}$ is based on the minimization of the bit cost (BC) for the transmission of the VQ error:

$$BC = -\sum_i P_i \log_2 Q_i . \quad (10)$$

The histogram of the data, e_{VQ} coming from a set of images outside the training set of the VQ codebook, are very well approximated by a Laplacian model with a proper value for σ . The BC reaches almost the entropy of the data $H_{e_{VQ}} = 3.74$ [bit/spl]. The standard



deviation σ is computed over an average histogram of some images. In fact other images give similar σ for the VQ error, and the bit cost measure is, in a relatively large neighborhood around the optimum σ , not highly sensitive to this parameter.

6 Simulation results

In this section, results of the described coding system are reported. All the simulations have been carried out on different still images (256 x 256, 8 bits gray levels). Symmetry extraction and polynomial approximation define the quality of the reconstructed image. The remaining coding process does not introduce any further distortion. The final bitrate R_{FINAL} is given by the luminance sample rate R_{SYM} of the polynomial approximation, divided by the compression ratio C_{EC} reached by the entropy coding, and the relatively small rate R_{PAI} of the PAIs coding.

$$R_{FINAL} = \frac{R_{SYM}}{C_{EC}} + R_{PAI} \quad [\text{bit/pixel}]. \quad (11)$$

In Table 1, the PSNR results and the relative bit rates for different test images are reported. The original image *Lena* and the reconstructed version are shown in Fig. 4.

image	PSNR [dB]	R_{FINAL} [bit/spl]
<i>Lena</i>	27.10	0.57
<i>Face</i>	27.57	0.54
<i>Miss America</i>	27.03	0.53

Table 1: Simulation results for three different test images. Vector Quantization codebook size = 1024.



Figure 4: Left: original "Lena"; right: reconstructed image.

If compared with the coding strategy as in [2], the presented scheme allows improvement both in terms of compression ratio and image quality, due to the lossless entropy coding. The degradation of the reconstructed image is due to the polynomial approximation. It should be pointed out that, if compared with [6], the polynomial approximation is improved, since it is calculated on a smaller area. However, visible blocking

effects remain, together with linear prediction errors. These problems can be overcome by the development of a segmentation-oriented technique and by a higher-order linear predictor.

The performances of the entropy coder C_{EC} are related to the VQ codebook size selected. The larger the codebook size, the higher the compression ratio. This gain in performances is achieved at the expense of computational complexity. Unfortunately there is no way to define numerically the efficiency of an entropy coding scheme when the data source contains unknown statistical dependencies among its symbols. We can compare the performance of our method with the zero-order entropy of the luminance sample values x , where statistical dependencies are not considered, and with the first-order entropy of a first order DPCM scheme. These entropies have been estimated over 10 images. The zero-order entropy is 7.84 [bit/spl], and the first-order one is 4.95 [bit/spl]. From Table 1, it is seen that the scheme based on VQ and arithmetic coding overperforms PCM and DPCM entropy coding.

7 Conclusions

In this work, we presented a coding technique which suggests the use of symmetry and entropy coding with VQ and arithmetic coding to reduce the redundancy in images. Coding by symmetries is a novel and open area of research; several aspects of the proposed system and its possible extensions are currently under investigation. In particular, the extension of this technique for a symmetry-based image segmentation system combined with higher-order linear prediction will be the object of future research.

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