

## Mirror Vision - Novel Approach to Obtaining 3-D Surface from One Single Image

Jun SHEN

Laboratoire Images, Institut de Géodynamique,  
Bât. Géologie, Av. des Facultés, Domaine Universitaire, 33405 Talence, France

RÉSUMÉ

*Dans cet article, nous proposons une nouvelle approche appelée "la vision par miroir" pour obtenir des données 3-D à partir d'une seule image à l'aide d'un miroir. Nous présentons d'abord le principe de cette méthode et analysons sa performance. Nous présentons ensuite une approche efficace "la vision par bi-miroir" pour résoudre l'ambiguïté de mise en correspondance. Nous terminons l'article en donnant l'implantation d'un tel système et la conclusion.*

ABSTRACT

*In the present paper, we propose a novel approach called mirror vision to obtain 3-D data from one single image in help of a mirror. The principle of mirror vision is first presented and the performance of mirror vision system is analyzed and compared with stereo vision. Bi-mirror vision is then presented as an efficient tool to solve false targets. A mirror vision system is briefly presented and some concluding remarks are given.*

### 1. INTRODUCTION

3-D data acquisition from 2-D images has been very important in image analysis and computer vision and many approaches have been proposed, such as photometric methods and stereo vision [1-7].

In the present paper, we propose a novel approach, that we call "mirror vision", to obtain 3-D data from a single image. Different from stereo vision technique using always more than one image, a mirror vision system uses only one image in help of a mirror. It is also distinguished from model based 3-D vision systems using a priori known 3-D object models.

This paper is organized as follows: in Section 2, we present the principle of mirror vision and show how the 3-D data can be found from a single image by use of mirror vision; in Section 3, the performance of a mirror vision system is analyzed; in Section 4, the bi-mirror vision is presented as an efficient tool to solve false targets; and in Section 5, a brief presentation of a mirror vision system and some concluding remarks are given.

### 2. PARALLEL MIRROR VISION SYSTEM

In this section, we present at first what a mirror vision system means and then show the principle of such a system for 3-D data calculation.

Without loss of generality, we can analyze mirror vision by use of a parallel system shown in Fig.1. A parallel vision system is composed of a camera and a mirror in the 3-D world, the optic axis of the camera is parallel to the mirror plane and the horizontal axis (or the vertical axis) of the 2-D coordinate system of the camera image plane is perpendicular to the mirror plane.

In Fig.1, OXYZ is the 3-D coordinate system in 3-D world, taking O as the origin and with axis OZ parallel to the camera optic axis and to the mirror surface M which is the plane YOZ. The optic center of the camera is at  $(-D, 0, 0)$  with  $D > 0$ , the center of the image  $o_1$  at  $(-D, 0, f)$ , where  $f$  is the focal distance of the camera, and the horizontal and vertical axes  $u$  and  $v$  of the image plane take respectively the direction of the X and Y axes of the 3-D world coordinate system.



We call such a system a parallel mirror vision system.

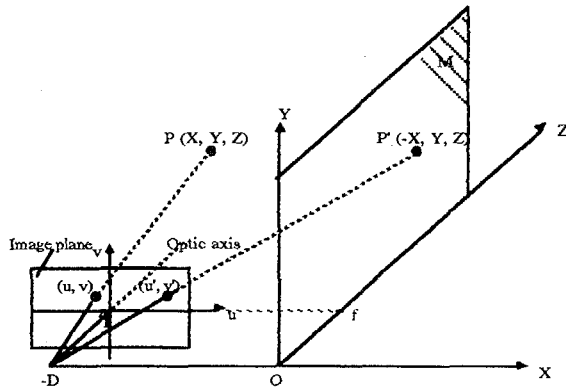


Fig.1. Parallel mirror vision system for 3-D data acquisition from a single image

Let  $P(X, Y, Z)$  be an object point in 3-D space on the same side of the mirror than the camera (otherwise this point would be hidden by the mirror and could not be seen by the camera). According to geometrical optics, a virtual image  $P'(-X, Y, Z)$  of the point  $P$  will be produced by the mirror. Let  $(u, v)$  and  $(u', v')$  be respectively the images of  $P$  and  $P'$  taken by the camera. By use of 3-D geometry, we have

$$(X+D)/u = Y/v = Z/f \quad (1)$$

$$\text{and } (-X+D)/u' = Y/v' = Z/f \quad (2)$$

Eqs. (1) and (2) give:

$$v = v' \quad (3)$$

$$\text{and } X = D(u - u') / (u + u')$$

$$\begin{cases} Y = 2Dv / (u + u') \\ Z = 2Df / (u + u') \end{cases} \quad (4)$$

where  $f$  is the focal distance of the camera and  $D$ , the distance from the camera optic center to the mirror, as is mentioned above.

Eq.(4) shows that the position of the object point  $P$  in the 3-D space can be uniquely determined from the 2-D image coordinates  $(u, v)$  and  $(u', v')$  of the corresponding pixels if the parameters of the camera and relative position of the camera and the mirror are known. Moreover, Eq.(3) shows that the image pixels, taken by the camera, of a point in 3-D space and its virtual image produced by the mirror have the same vertical coordinate on the image plane, which gives a strong geometrical constraint in corresponding pixels searching of a mirror vision system.

In practical applications, one may like to use a

convergent system, i.e., a one in which the camera optic axis composes an acute angle with the mirror plane in order to adapt the camera vision field to the objects interested. In this case, the same principle applies because all convergent mirror vision systems can be easily transformed to a parallel one by a camera rotation, and the image taken by this "virtual camera" after the rotation is uniquely determined from the image taken by the real camera of the convergent system [7]. Because of limit of space, we can not give more details here.

The relation between the point in 3-D space, the image of itself and that of its virtual image produced by the mirror is fundamental in mirror vision, from which geometrical constraints about corresponding primitives (sets of pixels) can be deduced. We would not discuss this problem in the present paper because it is another important problem beyond the scope of a 4 pages' paper. Similarly, the calibration problem to find the intrinsic and extrinsic parameters of a mirror vision system would not be discussed.

### 3. PERFORMANCE ANALYSIS

In Section 2, a new approach for 3-D data acquisition from a single image was proposed. Now we analyze the performance of such an approach.

First, we analyze the precision of a mirror vision system. A typical indice of precision is the precision of the depth  $Z$  calculated. Differentiating Eq.(4) with respect to  $(u + u')$ , we have

$$\partial Z / \partial (u + u') = -2Df / (u + u')^2 \quad (5)$$

$$\text{and } \partial Z / \partial (u + u') = -Z^2 / (2Df) \quad (6)$$

Eqs.(5) and (6) give the relative error of the depth  $Z$ :

$$|\Delta Z / Z| \approx |\Delta(u + u')| / Z / (2Df) \quad (7)$$

$$\text{and } |\Delta Z / Z| \approx |\Delta(u + u')\%| \quad (8)$$

where  $\Delta(u + u')$  is the error of the image matching result  $(u + u')$  and  $\Delta(u + u')\% = \Delta(u + u') / (u + u')$ , the relative error.

From Eq.(7), we see that for a fixed image matching precision, the relative error of the depth calculated from a mirror vision system is proportional to the depth of the point in 3-D world, the farther is

the point from the camera, the more important the relative error will be, and it can be improved by increasing the distance between the camera and the mirror. Eq.(8) shows that the relative precision of the depth calculated is equal to that of image matching result.

Now we compare the precision of a mirror vision system with that of a binocular stereo vision system. As is well known, the relative error of the depth calculated from a parallel binocular stereo vision system is:

$$|\Delta Z / Z| \approx |\Delta d / Z| (D_s / f) \tag{9}$$

where  $\Delta d$  is the error of image matching result (the precision of the disparity) and  $D_s$ , the distance between the two cameras. Comparing Eqs.(7) and (9), we see that in a limited space, if  $D = D_s$  and the same cameras are used, a mirror vision system can have a double precision than the conventional stereo vision techniques.

Another important indice of performance is the efficiency of the method. For mirror vision and stereo vision, the essential problem is the matching to find the correspondance between pixels, because false matchings will produce false targets.

We propose the following model to analyze the false target problem:

The image is composed of  $n$  lines and  $n$  coloums. The matching is done by use of feature pixels on the image, for example, the edge pixels. According to the analysis in Section 2, for a parallel or slightly convergent mirror system, the corresponding pixels will lie on the same line. We suppose that both the feature points in 3-D space and their virtual images produced by the mirror are seen by the camera and the probability that we have a feature point at a position in 3-D space is independent of that at other positions. The feature point density in the vision field is  $b$ .

Let  $P$  be a feature point in the 3-D space,  $P'$ , its virtual image, and  $p$  and  $p'$ , the image pixels projected respectively by  $P$  and  $P'$ . Obviously,  $p$  and  $p'$  are corresponding pixels by definition. So if there are only two feature pixels on the line to which a feature pixel  $p$  belongs, they will be  $p$  and  $p'$ , i.e., there would be no problem to match  $p$  and  $p'$ ,

therefore no false targets would occur. But if there exist more than 2 feature pixels on the line, the false targets do occur and they should be removed by some supplementary similarity constraints. So we can calculate the probability that we have false targets in a mirror vision system if no similarity constraints are used to remove them. As there are  $n$  lines on the image, the probability that the image of a feature point in 3-D space does not lie on the line of the pixel  $p$  will be  $(1 - 1/n)$ . No false targets would occur if and only if no one of the  $(n^2b - 1)$  feature points other than  $P$  projects its image on the line. So the probability that we have no false targets in a mirror vision system when no similarity constraints are used is:

$$P_{v2} = (1 - 1/n)^{n^2b - 1} \tag{10}$$

Because the edge density in real images is in general much smaller than 1 and the size  $n$  of the image, much larger than 1, Eq.(10) gives from Poisson's theorem:

$$P_{v2} \approx (1 - b)^n \tag{11}$$

Comparing this probability with the performace of a binocular stereo vision system [7], we see that a mirror vision system is as efficient as a binocular stereo vision system from the viewpoint of false targets. Anyway, this probability is very small, that is why all binocular stereo vision systems have to use some supplementary similarity constraints for solving false targets. Eq.(11) shows that this is true for mirror vision as well.

#### 4. BI-MIRROR VISION

Of course, we can use similarity constraints for image matching in a mirror vision system because in most cases, the image of a 3-D object should be similar to the symmetry (with respect to the vertical axis  $v$ ) of the image (taken by the camera) of its virtual image produced by the mirror. Such a system will be presented in Section 5.

To overcome the difficulty that some similarity constraints must be used for image matching, another strategy can be used, which is the bi-mirror vision.

A bi-mirror vision system is presented in Fig.2, where we have added "Mirror 2" in the system of Fig.1, and Mirror 2 is parallel to the plane XOZ,



i.e., perpendicular to Mirror 1 and to the camera image plane.  $P$  is a point in 3-D space;  $P'_1$  and  $P'_2$ , virtual images of  $P$  produced by the mirrors 1 and 2;  $(u, v)$ ,  $(u', v')$  and  $(u'', v'')$  denote respectively the coordinates of images of  $P$ ,  $P'_1$  and  $P'_2$  projected on the camera images plane.

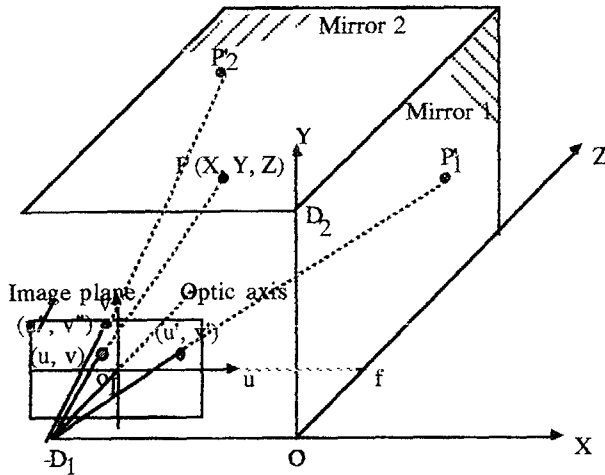


Fig.2. A bi-mirror vision system

Because of limit of space, we just explain the principle of the bi-mirror system and give directly the conclusion of analysis. For a feature pixel  $(u, v)$  on the image, we try to match the feature pixels lying on the same line with  $(u, v)$ . Taking a candidate feature pixel  $(u', v')$  on the line, if it is the corresponding pixel of  $(u, v)$ , we can determine a feature point  $P$  in 3-D space, and  $P$  should give in turn a virtual feature point  $P'_2$  produced by Mirror 2. This means that we must find a feature pixel at  $(u'', v'')$  which is the projection of  $P'_2$  on the image plane. The false targets analysis becomes thus much simpler: if we do find a feature pixel at  $(u'', v'')$ , we can accept the matching between  $(u, v)$  and  $(u', v')$ ; otherwise, the hypothesis of matching  $(u, v)$  to  $(u', v')$  should be rejected. By use of the same model in Section 3, a similar but more complicated statistical analysis shows that the false target probability for a bi-mirror vision system is practically zero if we match feature primitives (set of feature pixels), even when no a priori similarity constraints are used. Note that the analysis gives a strong theoretical support to the use of bi-mirror systems that solves much more easily the difficult false target problem in passive vision systems.

## 5. IMPLEMENTATION AND CONCLUSION

A mirror vision algorithm is implemented and tested, and interesting experimental results are obtained, which will be presented by slides because of limit of space. Edge pixel sequences were used as feature primitives and matching was done by use of edge sequence adjacency graph.

In conclusion, an original 3-D acquisition technique from a single image, i.e., mirror vision, is presented, which shows the following advantages:

- (1) Only one single 2-D image is needed to determine 3-D data and no a priori knowledge is necessary.
- (2) A double precision can be obtained with respect to conventional stereo vision techniques.
- (3) For bi-mirror vision systems, the false target probability is practically zero. Therefore the 3-D data obtained are very reliable.

Note that there exist many potential applications of the novel technique, such as in automatic assembly, quality control, particle tracing, motion analysis and other indoor and outdoor scene analysis.

## REFERENCES

- [1]. B.K.P. Horn, Understanding image intensities, *Artificial Intelligence*, 8, pp201-231, 1977.
- [2]. R. Bajcsy, Three-dimensional scene analysis, *Proc. of the 5th ICPR*, pp1064-1074, 1980.
- [3]. K. Ikeuchi & B.P.K. Horn, Numerical shape from shading and occluding boundaries, *Artificial Intelligence*, Vol.17, pp141-184, 1981.
- [4]. R.A. Jarvis, A perspective on range finding techniques for computer vision, *PAMI*, Vol.5, N°2, pp122-139, 1983.
- [5]. D. Marr & T. Poggio, A computational theory of human stereo vision, *Proc. Royal Soc.*, Vol.B204, pp301-328, 1979.
- [6]. W.E.L. Grimson, *From images to surfaces*, MIT Press, 1981.
- [7]. J. Shen, S. Castan & J. Zhao, A new passive measurement method by trinocular stereo vision, *Industrial Metrology*, Vol.1, N°3, pp231-259, 1990.