

DISTRIBUTED CFAR DETECTION IN WEIBULL CLUTTER

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Résumé

On considère un système qui utilise des décisions décentrés et qui travaille dans un clutter de Weibull dont les paramètres sont inconnus. Chaque radar emploie un algorithme à vraisemblance maximum pour ces décisions. La règle meilleure de fusion "K sur N" et les seuils locaux sont obtenus avec le test de Neyman-Pearson. On détermine la puissance du test en fonction du SCR de chaque récepteur, le nombre de radars et la probabilité globale de fausse alarme. On a traité les résultats théorique et des simulations.

1 - Introduction

In distributed radar systems with decentralised detection, built up of a set of radars distributed over a geographic area, a fusion center and a control center, each radar takes a binary decision about the presence or the absence of the target in the resolution cell under test independently from the others, and sends its partial result to the fusion center that takes the final decision. The management center controls the sensors and the fusion center in order to maximise the global system detection performance [1]. The benefits derived from a multiradar detection system are a broader coverage area, accuracy and reliability. The classical Neyman-Pearson criterion can be generalised to the distributed detection problem [2]. For a given system topology, the goal is to maximise the overall probability of detection at the fusion center ($P_{D_{tot}}$) while keeping the overall probability of false alarm ($P_{FA_{tot}}$) constant. In the case of parallel fusion network and assuming independent sensor observations with known distributions, the globally optimal structure consists of likelihood ratio tests at all sensors and a Neyman-Pearson test at the fusion center. Unfortunately the optimal thresholds of each sensor and fusion center are solutions of a set of coupled non-linear equations. So their evaluation is computationally difficult, particularly with a large number of sensors. In this work we assume the K-rank fusion rule: a target is declared to be present in the tested cell if at least K of the N detectors have made the same decision. The rank K spans from 1 to N, with K=1 and K=N corresponding to the Boolean "OR" and "AND" fusion rules. In such a framework the decentralised constant false alarm rate signal detection has been developed in the literature assuming Rayleigh-distributed clutter [3]. In realistic radar

Abstract

A distributed system with decentralised decisions, operating in Weibull clutter with unknown parameters is considered. Each sensor utilises a maximum likelihood CFAR algorithm for its decision. The optimum "K of N" fusion rule and the local thresholds of each receiver are obtained based on the Neyman-Pearson test. The power of the test is determined as a function of the signal to clutter ratios at each receiver, the number of radars and the overall false alarm probability. Theoretical and simulation results are discussed.

applications clutter distribution deviates quite often from Rayleigh, particularly when high resolution radars operate at low grazing angles. Weibull distribution fits well the data acquired from various types of clutter environments. Therefore our work is devoted to analyse the decentralised CFAR detection in Weibull clutter.

We assume, for the sake of simplicity, that the CFAR detectors have the same structure, while the signal to clutter ratios (SCR) for each sensor can be different.

2 - ML estimation of Log-Weibull parameters

CFAR (Constant False Alarm Rate) technique is a signal processing technique used in automatic detection radar systems to control the false alarm when the clutter parameters are unknown or slowly time-varying. The CFAR process adjusts detection threshold on a cell-by-cell basis so that, in clutter or noise interference environments, the false alarm probability remains constant. Assuming that the envelope detector output X is feed in a log amplifier the Weibull variate [4] is transformed in a Gumbel variate:

$$f_Y(y) = \frac{1}{b} \exp\left(\frac{y-a}{b}\right) \exp\left[-\exp\left(\frac{y-a}{b}\right)\right] \quad (1)$$

$$-\infty < y < \infty$$

where $-\infty < a < \infty$ is the location parameter, $b > 0$ is the scale parameter [4]. The expected value and the variance of the Gumbel distribution are:

$$E\{Y\} = a - \gamma b \quad \text{and} \quad \text{Var}\{Y\} = \frac{\pi^2}{6} b^2 \quad (2)$$

where γ is the Euler's constant ($\cong 0.5772$).

The detection strategy for target detection is:

$$\begin{aligned} Y_{cur} \geq \hat{T} & \quad \text{Target present} \\ Y_{cur} < \hat{T} & \quad \text{Target absent} \end{aligned} \quad (3)$$



where Y_{CUT} is relative to the cell under test (CUT) and the threshold \hat{T} is updated according to the rule:

$$\hat{T} = \hat{a} + g\hat{b} \tag{4}$$

The false alarm probability is given by:

$$P_{FA} = E_{\hat{T}} \left\{ \Pr(Y_{CUT} > \hat{T} | H_0) \right\} \tag{5}$$

As a consequence our aim is to develop a maximum likelihood (ML) CFAR algorithm for the Gumbel variate and to derive its performance by Monte Carlo simulations [5]. The ML estimates are obtained from $M/2$ samples leading the CUT and $M/2$ samples trailing the CUT (Fig.1).

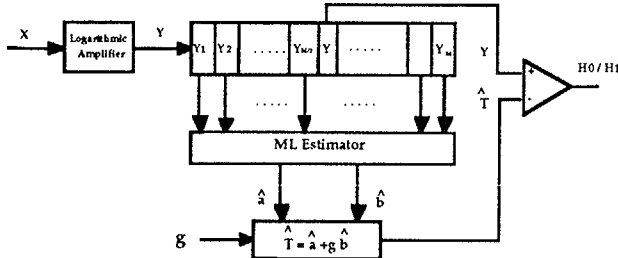


Fig. 1 ML-CFAR detector

If the M samples are assumed independent and identically distributed (i.i.d.) the ML estimators can be easily derived. They are given by:

$$\hat{b} = \frac{\sum_{j=1}^M y_j \exp\left(\frac{y_j}{\hat{b}}\right)}{\hat{b}} - \frac{1}{M} \sum_{j=1}^M y_j \tag{6}$$

$$\hat{a} = -\hat{b} \ln \left[\frac{M}{\sum_{j=1}^M \exp\left(\frac{y_j}{\hat{b}}\right)} \right] \tag{7}$$

Eqn.6 must be solved iteratively to yield \hat{b} , which will be used in eqn.7 to yield \hat{a} .

In order that the CFAR property holds the coefficient g in eqn.4 must be determined so that the prefixed P_{FA} results. The problem could be solved if the pdf $f_{\hat{T}}(t)$ were known but, unfortunately, we cannot find an analytic expression. However we observe that the joint pdf of \hat{a} and \hat{b} is asymptotically normal because \hat{a} and \hat{b} are maximum likelihood estimators and the pdf of \hat{T} is asymptotically normal too. We make use of these properties even for finite M . These approximations have been verified by Monte Carlo simulations: they are quite accurate for $M \geq 32$ and $P_{FA} \geq 10^{-7}$.

The expected value and the variance of \hat{T} are then:

$$E\{\hat{T}\} = a + gb \quad (\text{unbiased estimators}) \tag{8}$$

$$\text{Var}\{\hat{T}\} = \sigma_a^2 + g^2 \sigma_b^2 + 2\rho g \sigma_a \sigma_b \tag{9}$$

where $\sigma_{\hat{T}}^2$, $\sigma_{\hat{a}}^2$ and $\sigma_{\hat{b}}^2$ are the variances of \hat{T} , \hat{a} , \hat{b} respectively, g the threshold coefficient and ρ the correlation coefficient between \hat{a} and \hat{b} .

The Cramer-Rao lower bounds for σ_a^2 and σ_b^2 are:

$$\sigma_b^2 \geq \frac{6b^2}{M\pi^2} \quad \text{and} \quad \sigma_a^2 \geq \frac{6b^2}{M\pi^2} \left(1 + \frac{\pi^2}{6} + \gamma^2 - 2\gamma \right) \tag{10}$$

thus:
$$\sigma_{\hat{T}}^2 = \frac{k^2 b^2}{M} \tag{11}$$

with:

$$k^2 = \frac{6}{\pi^2} \left(1 + \frac{\pi^2}{6} + \gamma^2 - 2\gamma + g^2 + 2\rho g \sqrt{1 + \frac{\pi^2}{6} + \gamma^2 - 2\gamma} \right)$$

In these hypotheses P_{FA} is expressed by:

$$P_{FA} = \sqrt{\frac{M}{2\pi}} \frac{1}{k} \int_{-\infty}^{\infty} \exp(-h \exp y) \exp\left(-\frac{My^2}{2k^2}\right) dy \tag{12}$$

with $h \equiv \exp g$.

3 - Probability of detection and CFAR loss

In this section the probability of detection for Swerling type I target model is evaluated. Since the pdf of the clutter plus target cannot be obtained in closed form Monte Carlo simulation has been used. Fig.2 shows the results for $P_{FA} = 10^{-7}$ and $M=32$ reference cells, for several values of the shape parameter b .

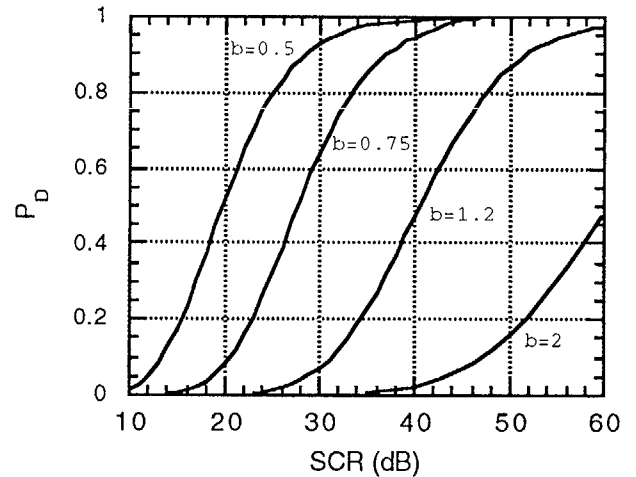


Fig.2 P_D curves ($P_{FA}=10^{-7}, M=32$)

As b gets greater than 0.5 (Rayleigh clutter), the clutter pdf has a longer tail, which produces an increase in adaptive threshold. This explains the decrease of P_D . It depends also from the reduced accuracy in estimating a and b . This effect is measured by the "CFAR loss" defined as the ratio between the signal to clutter ratio (SCR) required to achieve specified P_D and P_{FA} , and the SCR that would be necessary if the parameters a and b were known or perfectly estimated ($M=\infty$) and the threshold was a constant.

$$\text{CFAR}_{\text{loss}} = \frac{\text{SCR}(P_{FA}, P_D, b, M)}{\text{SCR}_{\infty}(P_{FA}, P_D, b)} \tag{13}$$

The CFAR losses plotted in Fig.3, represent the increase in signal power to obtain $P_D = 0.9$ in the non CFAR case.

The CFAR performance is represented in Fig.4 (for $M=32$ and $\text{SCR}=16$ dB as an example) in form of Receiver Operating Characteristics (ROCs) and for

several values of b . It is evident that the shape parameter b has a strong effect. The performance of CFAR receiver degrades as b changes from 0.5 to 2.

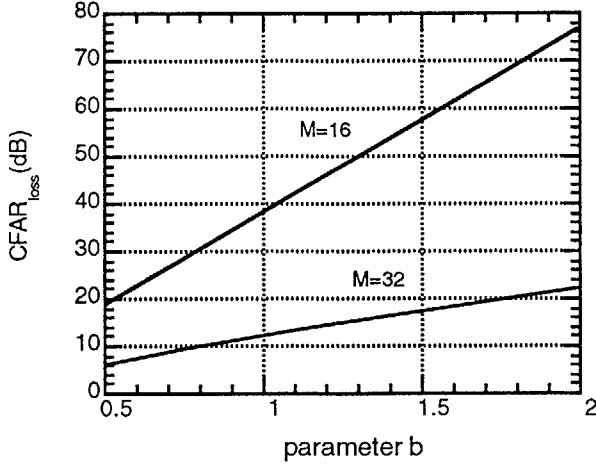


Fig.3 CFAR loss ($P_{FA}=10^{-7}, P_D=0.9$)

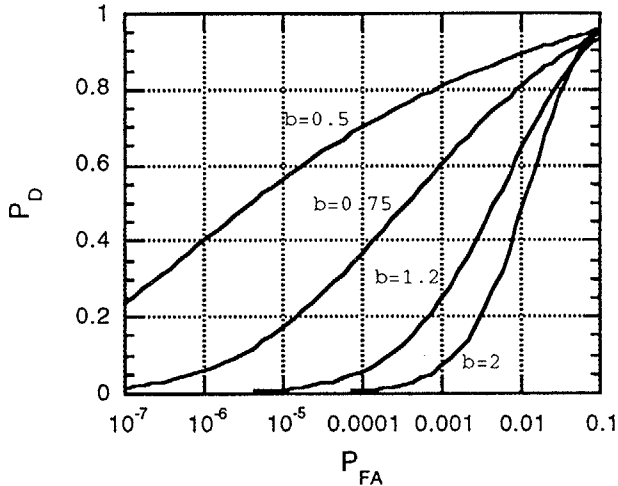


Fig.4 ROC curves ($SCR=16$ dB, $M=32$)

4 - Distributed CFAR detection

The overall probabilities of detection and false alarm at the fusion center may be written in similar expressions, given by:

$$P_{tot} = \sum_{u_1} \dots \sum_{u_N} R(u_1, \dots, u_N) \prod_{s_1} P_i \prod_{s_0} (1 - P_i) \quad (14)$$

where u_1, \dots, u_N are the local binary decisions and $R(u_1, \dots, u_N)$ is the Boolean function that corresponds to the fusion rule, that equals 1 if the overall decision u is for H_1 and H_0 otherwise. P_{tot} and P_i may represent the overall and local probability of detection or false alarm, s_1 is the set of the indexes i of the local decisions for H_1 , and s_0 the set of the indexes of the decisions for H_0 . Using the K -rank fusion rule eq. 14 can be expressed in recursive form [3]:

$$P_{tot} = \sum_{i=K}^N \left\{ \left(\sum_{p=0}^{i-K} (-1)^p \binom{i}{p} \right) \left(\sum_{q=1}^{N-i+1} P_q \left(\sum_{r=q+1}^{N-i+2} P_r(\dots) \right) \right) \right\} \quad (15)$$

Combining the ROCs with the relationship (15) we have at the fusion center:

$$P_{Dtot} = F_{N,K}(g_1, \dots, g_N, b, SCR_1, \dots, SCR_N) \quad (16)$$

$$P_{FAtot} = G_{N,K}(g_1, \dots, g_N)$$

where the form of the functions $F_{N,K}$ and $G_{N,K}$ depends on N and K .

Given the number N of radars in the network, and assuming known the shape parameter b and the SCR_i (i.e. the local ROCs), the goal is to find the rank K and the threshold coefficients g_i that maximise the P_{FAi} for a fixed P_{FAtot}^* at the fusion center. For the cell under test we can assume as clutter shape parameter the estimate provided by the local CFAR detectors, whereas the SCR_i can be calculated from the estimated clutter scale parameter. The problem can be solved computing for each $K=1, \dots, N$ the g_1, \dots, g_N that maximise P_{Dtot} for the given P_{FAtot}^* , and choosing as globally optimal solution the rank K and the threshold coefficients g_1, \dots, g_N that provide the largest P_{Dtot} .

For each K the problem to be solved is a non-linear constrained optimisation problem.

In the case of equal SCR_i the problem is greatly simplified if the g_i are forced to be equal. This condition proved itself to be optimal for Rayleigh fluctuating targets. In such a case all the P_{Di} and P_{FAi} are equal, and eq. 14 assumes a simple binomial expression:

$$P_{tot} = \sum_{i=K}^N \binom{N}{i} p^i (1-p)^{N-i} \quad (17)$$

where P is the common P_i . The optimisation program inputs are the number of sensors, the overall probability of false alarm, and the local ROCs determined in tabular form. The outputs are the overall probability of detection for the N rank fusion rules, the optimal rank and the corresponding threshold coefficients, all functions of the examined SCRs.

5 - Results and discussion

Several situations using networks with ML-CFAR detectors have been studied according to the proposed method assuming Swerling targets and Weibull clutter. As expected, the resulting overall power of the test (i.e. P_{Dtot} for a fixed P_{FAtot} as a function of the SCRs) increases with the number of radars. When the SCRs are equal, inverting the power of the test curves for a given P_{Dtot} , we get the SCR necessary at each sensor to obtain the wanted P_{Dtot} as a function of the number of sensors N . The results are shown in Fig.5 for $P_{Dtot} = 0.9$, $P_{FAtot} = 10^{-7}$, $M=32$.

The figure shows clearly the network gain, i.e. the decrease of SCR necessary to obtain the wanted performance, that yields increased coverage area. When the number of radars exceeds 5 or 6, the addition of a new radar does not significantly improve the detection performance of the network, especially in the Rayleigh case. It is therefore necessary a compromise



between performance and network cost.

Interestingly, network gain is higher in spiky clutter and system performance for large N overcomes the Rayleigh one.

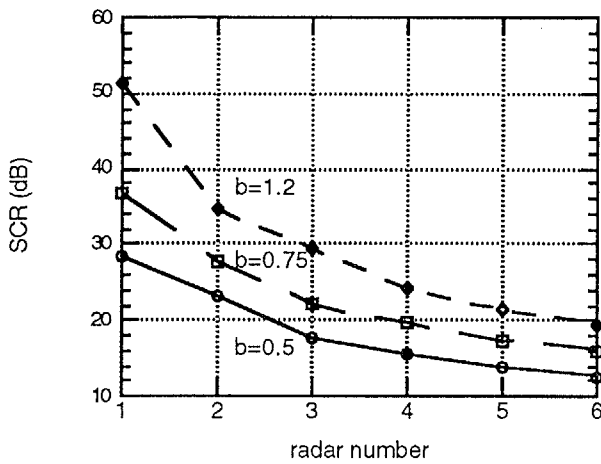


Fig.5 ($P_{FA_{tot}}=10^{-7}, P_{D_{tot}}=0.9, M=32$)

Network $P_{D_{tot}}$ is less sensitive to shape parameter b than a single radar. In order to evaluate the network ML-CFAR loss, network performance has also been calculated for $M=\infty$. The loss corresponding to a single radar (fig. 3) decreases very quickly as the sensors number increases, and for $N>4$ it is nearly independent from b and lower than 2 dB. These results, together with Fig.5, allow to achieve a trade-off between number of radars and complexity of the ML-CFAR processors. When the number of radars is large it is not necessary to use detectors with a large number of reference cells.

For different SCRs we report for simplicity the case $N=2$. Obviously system performance will be better than that achievable with the less noisy detector alone (i.e. the one with the higher SCR). Coverage area will be therefore wider than the union of the coverage areas of the two radars operating apart.

Fig. 6 shows the SCR necessary at one radar to obtain the wanted performance as a function of the SCR at the other radar, for the same $P_{D_{tot}}, P_{FA_{tot}}, M$ and b as above. We remark that the most noisy detector (i.e. the one with the lower SCR), although its performance is very poor when operating alone, contributes significantly to system performance when co-operating with the other detector, especially for the case of spiky clutter. Obviously when the lower SCR goes to $-\infty$ (dB) performance reaches that of the other radar alone

6- Conclusions

The optimum value of rank K and the corresponding operating points of the local detectors, for equal SCRs exhibit the following trends:

- 1) The optimum K starts from the maximum value N ("AND" rule) for low values of SCR, and decreases as SCR increases, towards K=1 ("OR" rule).
- 2) For increasing shape parameter b and/or decreasing $P_{FA_{tot}}$, the optimum ranks tend to be higher
- 3) In the Rayleigh case the performance achievable with the various rank fusion rules are less dispersed and less sensitive to SCR changes than in spiky clutter.

4) For large N, spiky clutter, and low SCR, the optimal K is high and the corresponding operating points of the local detectors are characterised by low threshold coefficients and high values of P_{FA} . In such a case the high average data rate can overcome the communication channel capacity, and it may be suitable to choose a K lower than the optimum to get more reasonable operating points.

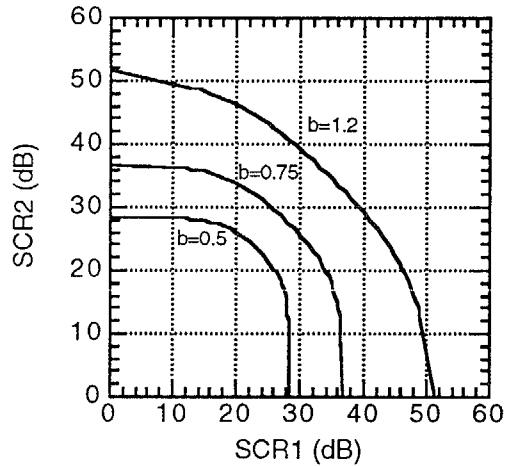


Fig. 6 ($P_{FA_{tot}}=10^{-7}, P_{D_{tot}}=0.9, M=32$)

For different SCRs and $N=2$, the following remarks can be made:

- 1) The optimum rank in the examined ranges of SCRs is always $K=2$ except for the Rayleigh case where K becomes 1 when one or both SCRs are high.
- 2) The optimisation yields a true global optimum, since for $N=2$ the examined rules ("AND" and "OR") are all the possibly optimal fusion rules.
- 3) In the AND case, as the SCRs difference increases, the optimal threshold coefficient of the most noisy detector (i.e. the one with the lower SCR) decreases whereas the one of the less noisy detector i increases (a dual behaviour arises operating with the "OR" strategy). As a limit, the "AND" decision of the fusion centre just sets $u=u_i$, so all decision making is really done by the detector with higher SCR. Approaching this situation, the most noisy detector is no longer really useful and its data rate may overcome channel capacity, and can be therefore excluded from the decision process.

7- References

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