AN APPLICATION OF THE WIGNER-VILLE DISTRIBUTION FOR THE INITIALIZATION OF ISAR IMAGE AUTOFOCUSING TECHNIQUES

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RÉSUMÉ

La méthode de la compensation du mouvement est l'un des pas critiques de la reconstruction de l'image de l'ISAR et elle est appliquée en utilisant les technique de l'autofocalisation. Ces techniques ont besoin d'un vecteur d'initilisation dont les composantes représentent une estimation des paramètres du mouvement du target. Le choix de ce vecteur d'initilisation est très critique parce que ce choix détermine la convergence de l'algorithme de l'autofocalisation. Dans cet article on propose une méthode nouvelle par l'estimation du vecteur d'initialisation qui se base sur l'utilisation de la distribution de Wigner-Ville (WVD) avec la technique de la décomposition en multiresolution.

ABSTRACT

The motion compensation procedure is one of the critical step of ISAR image reconstruction and it is usually performed by using autofocusing techniques. These autofocusing techniques need an initialization vector whose components represent a raw estimation of the target motion parameters. The choice of this initialization vector is a critical point since it determines the convergence of the autofocusing algorithms. In this paper we propose a novel method for estimating the initialization vector based on the use of the Wigner-Ville Distribution (WVD) together with the multiresolution decomposition technique.

1. INTRODUCTION

The Inverse Synthetic Aperture Radar Imaging (ISAR) is a technique that provides high resolution radar images of objects. High cross-range resolution is obtained by a coherent processing of radar returns received for different aspect angles of the object with respect to the radar and high range resolution is achieved by using large bandwidth transmitted signal. One of the main and critical step of the ISAR technique is the motion compensation that is the operation necessary for focusing the reconstructed image. The compensation consists in removing from the received signal a phase term depending on the motion of the target and to this purpose accurate estimates of the target motion parameters are required. Tracking data or instrumentation data, as in SAR (Synthetic Aperture Radar) case, cannot be used because of their low accuracy. Hence the motion compensation is usually performed by autofocusing techniques which provides accurate estimates of the target motion parameters by means of the samples of the received signal. All the autofocusing technique proposed in literature are based on a maximization of a functional of the reconstructed image (the image absolute maximum value, the image contrast, range profile cross-correlation). The numerical methods used for maximizing the functional requires a suitable initialization vector to be performed. The choice of this initialization vector (the components of this vector are a raw estimates of the target motion parameters) is a critical point since it determines the convergence of the autofocusing algorithms. In this paper a novel method for estimating the initial target motion parameters that guarantee the convergence of different autofocusing algorithms is proposed. The method is based on the evaluation of the Instantaneous signal Frequency (IF) as the mean conditional frequency of the Wigner-Ville Distribution (WVD) [1] of the received signal. The components of the initialization vector are evaluated from the coefficients of the regression line of the IF multiresolution decomposition [2] at a given resolution level. In next sections the image focusing problem is briefly recalled and a detailed description of the proposed method is provided. Some simulation results are presented and discussed in section 4.

2. THE RADAR IMAGE FOCUSING PROBLEM

Referring to Fig.1 and following the Walker approach [3] the Fourier Transform (FT) of the analytic signal associate with the signal received on the k-th sweep can be written as:

$$S_R(f,kT_R) = S_T(f)\ \exp\{-j2\pi f\,\frac{2R_0(kT_R)}{c}\}$$

$$\int_V f(x)\ \exp\{\,j\,2\pi xX\}\ dx \qquad 0 \le k \le M-1 \quad (1)$$

where

$$\mathbf{X}(f,kT_R) = -\frac{2f\,\mathbf{i}_{R_0}(kT_R)}{c} \qquad (f_0 - \frac{B}{2} \leq f \leq f_0 + \frac{B}{2}) \quad (2)$$

where T_R is the pulse repetition interval, M the number of sweeps, f_0 the carrier frequency, $S_T(f)$ the Fourier transform of the transmitted signal $s_T(t)$ of bandwidth B, $R_0(kT_R)$ the modulus of the vector $\mathbf{R}_0(kT_R)$ which represents the position of the origin of the system of coordinates fixed on the target with respect to the radar ad $\mathbf{i}_{R_0}(kT_R)$ its unit vector. The function f(**x**) in eq.(1) is called reflectivity function and it describes the backscattering properties of the object with respect to the system of coordinates fixed on the target. By removing the term $S_T(f)\exp\{-j2\pi f \frac{2R_0(kT_R)}{c}\}$ eq.(1) becomes the FT of the reflectivity function f(**x**) calculated on the values of **X** that satisfy the parametric equations (2). At this point an Inverse FT allows the reconstruction of the reflectivity function $\hat{f}(\mathbf{x})$ whose modulus represents the reconstructed image of the object. The procedure of removing the term $\theta(f,kT_R)=S_T(f)\ \exp\{-j2\pi f \frac{2R_0(kT_R)}{c}\}$ is called motion compensation and it is a fundamental step for achieving a correct focusing of the reconstructed image. The compensation is critical because we have not a priori knowledge of the $R_0(kT_R)$ term and in any case the range data measured by the radar system cannot be used for motion compensation because of their low accuracy. Hence, the estimation of $R_0(kT_R)$ can be only performed by using the samples of the received signal, i.e. by means of autofocusing techniques. In most practical cases the term $\exp\{-j2\pi f \frac{2R_0(kT_R)}{c}\}$ can be approximated by taking the Taylor series of the $R_0(kT_R)$ stopped at the second term:

$$\exp\{-j\frac{4\pi f}{c}[\alpha + \beta kT_R + \gamma(kT_R)^2]\}$$

with $\alpha=R_0(0)$, $\beta=\dot{R}_0(0)$, $\gamma=\ddot{R}_0(0)/2$. The phase term depending on the parameter α can be neglected since it produces only a shift of the reconstructed image without giving any unfocusing of the image Thus, only the estimation of β and γ are necessary for focusing the image. The autofocusing techniques provide accurate estimates of the target motion parameter β and γ organised in a vector by maximizing a functional of the reconstructed image. The maximization procedures requires an initialization vector whose components $\hat{\beta}$ and $\hat{\gamma}$ must be evaluated with an accuracy such that the convergence of the autofocusing algorithms is guaranteed. In the following it will described a method for calculating the initial vector $\mathbf{y}_0=[\hat{\beta}, \hat{\gamma}]$ by using the WVD of the signal together with the multiresolution decomposition technique.

3. INITIALIZATION OF IMAGE AUTOFOCUSING TECHNIQUES

Referring to Fig.1 let us consider a new Cartesian coordinate system (u_1,u_2) with the some origin of (x_1,x_2), and with the axis u_1 aligned with $\mathbf{R}_0(kT_R)$ at any instant kT_R. Substituting eq.(2) in eq.(1) we can note that in the argument of the exponential function within the integral, appears the inner product between $\mathbf{i}_{R_0}(kT_R)$ and **x** which is a scalar quantity $r(kT_R)$ representing the coordinate of the vector **x** along the u_1 axis.

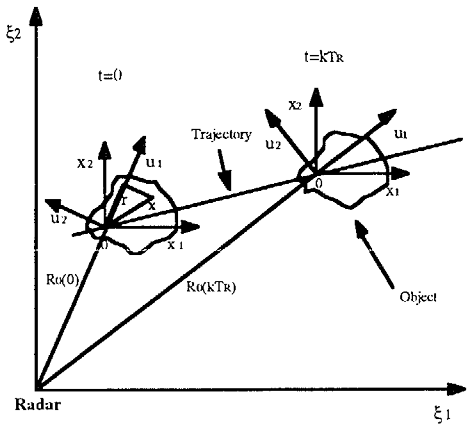

Fig.1 Reference system

By modelling the targets as a set of N ideal scatterers, the reflectivity function can be written as: $f(\mathbf{x}) = \sum_{i=1}^{N} D_i\ \delta(\mathbf{x}-\mathbf{x}_i)$ where $\mathbf{x}_i$ are the vector that locates the position of the i-th scatterer and D_i is its backscattering complex coefficient. Considering assumption above and taking only the carrier frequency of the spectrum eq.(1) becomes:

$$z(kT_R) = S_R(f_0,kT_R) = S_T(f_0)\ \exp\{-j\frac{4\pi f_0}{c}R_0(kT_R)\} \sum_{i=1}^{N} d_i\, e^{-j[(4\pi f_0/c)\, r_i(kT_R) - \varphi_i]} \quad (3)$$

where $r_i(kT_R)$ is the coordinate of the i-th scatterers on the axis u_1 and where the backscattering coefficients D_i have been expressed as $D_i=d_i\, e^{j\varphi_i}$. Considering the Taylor series expansion of $R_0(kT_R)$ stopped at the second term and assuming the approximation $r_i(kT_R)=a_i+b_i\, kT_R$ with $a_i = r_i(0)$ and $b_i = \dot{r}_i(0)$, the signal $z(kT_R)$ can be rewritten as

$$z(kT_R) = A\ \exp\{-j\frac{4\pi f_0}{c}[\beta\, kT_R + \gamma\,(kT_R)^2]\} \sum_{i=1}^{N} d_i\, e^{-j\{(4\pi f_0/c)\cdot[a_i+b_i\, kT_R] - \varphi_i\}} \quad (4)$$

where $A=S_T(f_0)\exp(-j\frac{4\pi f_0}{c}\alpha)$ is a complex constant.

By considering the discrete WVD of $z(kT_R)$ as defined in [4] :

$$W(n,m)= \sum_{k=0}^{M-1} z[(n+k)T_R]\, z^*[(n-k)T_R]\, e^{-j(2\pi/M)mk} \qquad (5)$$

and after some manipulations, the mean conditional frequency can be expressed by:

$$f_{CF}(kT_R) = -\frac{2f_0}{c}[\beta+ 2\gamma\, kT_R] - \frac{f_u(kT_R)}{f_d(kT_R)} \qquad (6)$$

where

$$f_u(kT_R)=\sum_{i=1}^{N} \frac{2f_0}{c} d_i^2 b_i + \frac{2f_0}{c}\sum_{i=1}^{N-1}\sum_{j=i+1}^{N} d_i d_j (b_i+b_j) \cos[\frac{4\pi f_0}{c}(a_{ij}+ b_{ij}\, kT_R) +\varphi_{ij}]$$

and

$$f_d(kT_R)=\sum_{i=1}^{N} d_i^2 + 2\sum_{i=1}^{N-1}\sum_{j=i+1}^{N} d_i d_j \cos[\frac{4\pi f_0}{c}(a_{ij}+ b_{ij}\, kT_R) +\varphi_{ij}]$$

where $a_{ij}=a_i-a_j$, $b_{ij}=b_i-b_j$ and $\varphi_{ij}=\varphi_i-\varphi_j$. For a property of the WVD [5] the function $f_{CF}(kT_R)$ represents the Instantaneous Frequency (IF) of $z(kT_R)$ calculated at the instants kT_R. As shown in eq. (8) the mean conditional frequency can be decomposed as a sum of two terms: the linear term $-\frac{2f_0}{c}[\beta+ 2\gamma\, kT_R]$ which contains the parameters β and γ we would have to estimate; the oscillatory term $\frac{f_u(kT_R)}{f_d(kT_R)}$ which represents a noise term in the estimation of the target motion parameters. The parameters β and γ can be extracted by applying a linear regression to the IF. This is not the best solution since several high peaks of the IF with the same polarity could affect the values of the coefficients of the regression line with a consequent wrong estimation of β and γ. An improvement of the method can be obtained by performing a multiresolution decomposition of the IF at a suitable level. As shown in [2], if the "scaling function" is a low pass signal, subsequent applications of the mutiresolution decomposition to a signal s(t) eliminate higher frequency components of the spectrum of s(t) so that sharper peaks are progressively reduced. Lemarie mutiresolution decomposition of the IF permits the attenuation of the sharpest spikes of the IF. After that, a linear regression is applied to extract the estimates $\hat{\beta}$ and $\hat{\gamma}$. Although the mutiresolution decomposition gives more accurate estimates these cannot be used for focusing directly the image. In any case $\hat{\beta}$ and $\hat{\gamma}$ can be successfully employed to initialize the autofocusing algorithms and to ensure their convergence. The method can be briefly summarized as follows:

1) Calculate the signal $z(kT_R)$ as the FT of the received data in correspondence with the carrier frequency for each sweep.
2) Evaluate the WVD of $z(kT_R)$ and calculate the mean conditional frequency $f_{CF}(kT_R)$.
3) Determine the sequence $f_{MR}(kT_R)$ via a multiresolution decomposition of $f_{CF}(kT_R)$;
4) Apply the linear regression to $f_{MR}(kT_R)$ in order to determine the regression line $f_{RL}(kT_R)=p\, kT_R + q$.
5) Calculate the initial vector $y_0=[\hat{\beta}, \hat{\gamma}]$ as follows:

$$\hat{\beta}=-\frac{c}{2f_0}q=-\frac{\lambda_0}{2}q \qquad \hat{\gamma}=-\frac{c}{4f_0}p=-\frac{\lambda_0}{4}p$$

4. SIMULATION RESULTS

In this section some preliminary results obtained by applying the proposed method on data produced by a simulation program are presented and discussed. As first case let us consider a target modelled as a set of ideal scatterers whose spatial position is referred to the system of coordinate $[x_1,x_2]$ parallel to the system of coordinate $[\xi_1,\xi_2]$ having origin in the radar (see Fig. 1). The target moves along a straight trajectory with a constant velocity v=800 Km/h at an initial distance of 6 Km and with the angle between the velocity vector and cross-range direction equal to 1°. The target is composed of five scatterers located at S_1≡(0m, 0m), S_2≡(10m, 5m), S_3≡(5m, 10m), S_4≡(-5m, 10m), S_5≡(10m, 5m) with backscattering coefficients real and equal to 1. The radar transmits 1024 pulses at a frequency of 10 GHz with a pulse repetition time T_R=400 µs. In this operating condition the cross-range resolution is about 1 m and the target motion parameter vector is **x**=[3.878 m/s, 4.114 m/s^2] (the components of this vector are β and γ). As shown in Fig.2 the WVD of the received signal is concentrated in a limited region following the linear term of the IF of the received signal. Fig.3 shows the mean conditional frequency extracted from the WVD. Several positive and negative peaks due to the oscillatory term of the IF are located around a straight line corresponding to the linear term of eq.(6). By applying a linear regression to the IF an estimated vector $\mathbf{y_0}^{(CF)}$=[3.639 m/s, 4.384 m/s^2] is obtained. As shown in Fig.4, an attenuation of the sharper peaks is achieved by means of a mutiresolution decomposition of the IF at resolution level of 16. After the linear regression the estimated vector becomes $\mathbf{y_0}^{(MR)}$=[3.738 m/s , 4.309 m/s^2]. The application of the mutiresolution gives a slight improvement of the results even if the vector $\mathbf{y_0}^{(MR)}$ cannot be directly employed for focusing the image since the error between the components of $\mathbf{y_0}^{(MR)}$ and **x** are too high for the purpose of ISAR image focusing.However, the errors are small enough for adopting the estimated vector for the initialization of the autofocusing techniques. Although in section 3 we have only considered the case of target composed of ideal scatterers, the method has been also validated by taking electromagnetic model of the target obtained according to the Physical Optic (PO). The geometrical surface of an MD87 aircraft has been approximated by using a flat plate model and Physical Optic Theory has been used for the received field prediction. The target moves along a circular trajectory of radius 1 Km at an initial distance of 13.5 Km and with a velocity of 120 m/s.

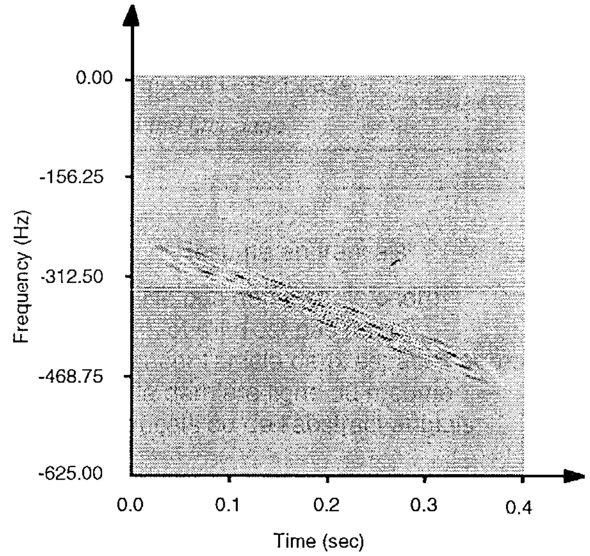

Fig.2 WVD of the received signal

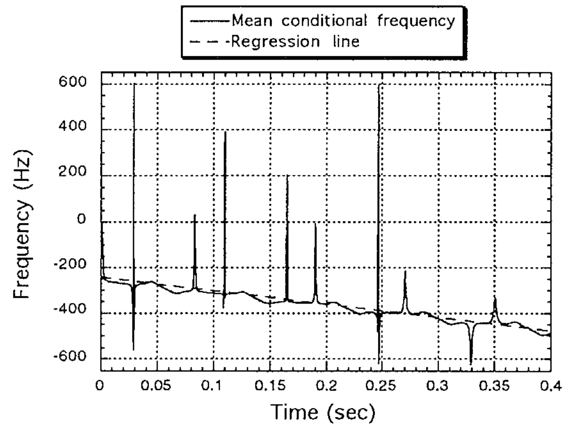

Fig.3 Mean conditional frequency extracted from the WVD

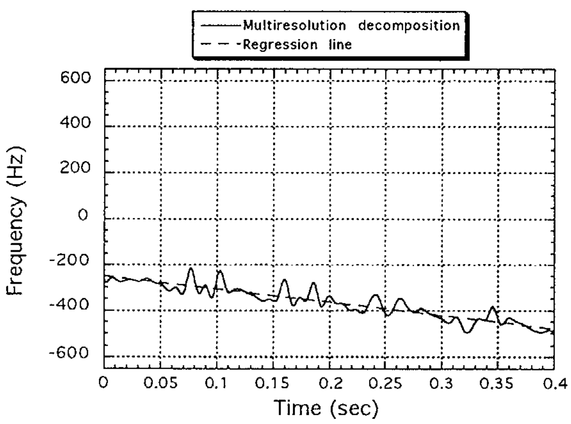

Fig.4 Mutiresolution decomposition of the signal plotted in Fig.3 and relative regression line

The angle between cross-range direction and velocity vector at the initial instant is about 30°. The radar transmits 1024 pulses at a frequency of 10 GHz with a pulse repetition time T_R=260 μs. The attainable cross-range resolution is about 0.5 m and the motion parameter vector is x=[40.621 m/s, -6.340 m/s^2]. By applying the method without multiresolution we have $y_0^{(CF)}$= [11.510 m/s, -5.950 m/s^2]. It is not surprising if $\hat{\beta}$ is much different than β because this is due periodicity of the WVD in frequency that causes an ambiguity on the measure of $\hat{\beta}$ of $\Delta\hat{\beta}=\lambda_0/(4\ T_R)$. The vector $y_0^{(CF)}$ after correction of the ambiguity becomes $y_0^{(CF)}$= [39.960 m/s, -5.950 m/s^2]. If we apply the method with a mutiresolution decomposition of resolution level 8, the estimated vector corrected of the ambiguity $y_0^{(MR)}$= [39.978 m/s, -6.050 m/s^2]. Also in this case there is a slight improvement on the final result by adopting the multiresolution step. The errors between the components of $y_0^{(MR)}$ and x are so small that the use of the method for the initialization of the autofocusing algorithms ensures the convergence of the algorithms itself.

5. CONCLUSION

A theoretical validation of the proposed method requires the analysis of the maximum acceptable errors on the estimates of β and γ. This analysis is quite complicated because of the functional of the autofocusing technique which are not always available in a closed form. Extensive simulations with different type of targets show that the convergence of the autofocusing algorithms is guaranteed when the accuracy on the estimates provided by our method is at least one hundredth of the accuracy required for a correct focusing of the ISAR images. All the results obtained by applying the proposed method for intialising the autofocusing technique proposed in [6] satisfy the condition above and ensure the convergence of the algorithm. However some aspects of this method are under investigation. A method for removing the problem of the ambiguous estimate of the radial velocity is under study. The behaviour of the proposed method in presence of noise is being analysed.

6. REFERENCES

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