On the MAC layer Optimal Power Allocation in NOMA Uplink Networks

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Résumé – Dans cet article, nous proposons des nouvelles expressions analytiques de la capacité effective (EC) des utilisateurs NOMA. Ensuite, l'analyse asymptotique des performances de NOMA et de OMA montre que le NOMA nécessite une bonne allocation de puissance pour être meilleur que le OMA dans les RSB (rapport signal-sur-bruit) faibles et moyens pour l'utilisateur faible U_1, et dans les RSB élevés pour l'utilisateur fort U_2. Nous proposons une politique d'allocation de puissance aux RSB extrêmes pour optimiser le NOMA, mais cela se fait au détriment de la réduction du délai dans les RSB élevés.

Abstract – In this paper, we propose new analytical expressions for the effective capacity (EC) of NOMA users. Then, the asymptotic performance analysis of NOMA and OMA shows that NOMA requires good power allocation to be better than OMA in low and medium SNRs (signal-to-noise ratio) for the weak user U_1, and in high SNRs for the strong user U_2. We propose a power allocation policy at extreme SNRs to optimize NOMA, but this comes at the expense of delay reduction in high SNRs.

I Introduction

Non-orthogonal multiple access (NOMA) schemes have attracted a lot of attention recently and has succeeded in becoming a promising candidate as a multiple access technique for future generations of wireless networks. The main interest of NOMA lies in the fact that it allows multiple users to be served simultaneously within the same resource with enhanced spectral efficiency [1]. It relies typically on the use of superposition coding at the transmitter and of the successive interference cancellation (SIC) at the receiver [2]. In the uplink NOMA networks, both are performed at the receiver, where the strong user's message is decoded first (as opposed to the downlink) and subtracted from the users' superimposed signal before the decoding of the weak user's message.

Besides, in ultra reliable low latency communications (URLLC), delay quality of service (QoS) becomes increasingly important. In fact, in future wireless networks, users are expected to necessitate flexible delay guarantees for achieving different service requirements. In order to satisfy diverse delay requirements, a simple and flexible delay QoS model is imperative to be applied and investigated. In this respect, the effective capacity (EC) theory can be employed. EC denotes the maximum constant arrival rate which can be served by a given service process, while guaranteeing the required statistical delay provisioning [3], [4], [5]. Delay-constrained communications for a downlink NOMA network are studied in [6] and with secrecy constraints in [7], [8].

In the present analysis, we focus on uplink transmissions; mainly because NOMA, as a spectrum-efficient multiple access technique, is considered to be promising for supporting the massive number of devices to access the uplink connections.

This work is based on [9], where we first derive novel closed-form expressions for the individual ECs in a two-user NOMA uplink network; then provide an asymptotic performance analysis to compare NOMA and the orthogonal multiple access (OMA) considering fixed power coefficients, and whose result is presented in the form of Lemma. The novelty here is that we provide, in the form of Propositions, the analytical expressions of the optimal power coefficients at Layer-2 or MAC layer that ensure NOMA users to have at least the OMA performance in low and high SNRs. All the proposed expressions are validated by an extensive set of simulations.

The rest of the paper is organized as follows. In Section II we present the system model. Section III presents the investigation of the EC in a two-user NOMA uplink network. Simulation results are given in Section IV, followed by conclusions in Section V.

II Uplink System Model

We assume a two-user NOMA uplink network with users U_1 and U_2 in a Rayleigh fading propagation channel, with respective channel gains during a transmission block denoted by $|h_1|^2 < |h_2|^2$. The users transmit corresponding symbols s_1, s_2 respectively, with $\mathbb{E}[|s_i|^2] = 1$.

The base station (BS) observes the following superimposed signal,

$$z = \sum_{i=1}^{2} \sqrt{\alpha_i P_t} h_i s_i + w, \quad i = 1, 2, \tag{1}$$

where w denotes a zero mean circularly symmetric complex Gaussian random variable with variance σ^2, i.e., $w \sim \mathcal{CN}(0, \sigma^2)$. P_t is the total power, while α_i is the NOMA power coefficient for $U_i, i = 1, 2$, with $\sum_{i=1}^{2} \alpha_i = 1$.

Following the SIC principle, the achievable rates, in b/s/Hz, for user $\mathrm{U}_i, i = 1, 2$, is expressed as [2]

$$R_i = \log_2 \left(1 + \frac{\rho \alpha_i |h_i|^2}{1 + \rho \sum_{l=1}^{i-1} \alpha_l |h_l|^2} \right), \qquad (2)$$

where $\rho = \frac{P_t}{\sigma^2}$ denotes the transmit SNR.

Applying the EC theory in a uplink NOMA with two users, the i-th user's EC over a block-fading channel, is defined as [3]

$$E_c^i = -\frac{1}{\theta_i T_f B} \ln \left(\mathbb{E} \left[e^{-\theta_i T_f B R_i} \right] \right) \quad (\text{in b/s/Hz}), \qquad (3)$$

where T_f is the fading-block duration, B is the bandwidth and $\mathbb{E}[\cdot]$ denotes expectation over the channel gains and θ_i is the statistical delay QoS exponent of the i-th user. In fact, smaller θ_i indicates that the system is more delay-tolerant, while a larger θ_i corresponds to a system with more stringent QoS requirements.

By inserting R_i into (3), we obtain the following expression for the EC of the i-th NOMA user

$$E_c^i = \frac{1}{\beta_i} \log_2 \left(\mathbb{E} \left[(1 + \frac{\rho \alpha_i |h_i|^2}{1 + \rho \sum_{l=1}^{i-1} \alpha_l |h_l|^2})^{\beta_i} \right] \right), \qquad (4)$$

where $\beta_i = -\frac{\theta_i T_f B}{\ln 2}$, $i = 1, 2$, is the normalized (negative) QoS exponent. Likewise, the EC of the OMA users is given as

$$\widetilde{E}_c^i = \frac{1}{\beta_i} \log_2 \left(\mathbb{E} \left[(1 + \rho |h_i|^2)^{\frac{\beta_i}{2}} \right] \right), \quad i = 1, 2. \qquad (5)$$

where the $\frac{1}{2}$ in the exponent corresponds to the half time resource sharing in OMA of which the tilde is a marker.

III Effective Capacity

III.1 Closed-form expressions

To obtain the closed-form expression for ECs, we first make use of the theory of order statistics [9] [10] to find the probability density function (PDF) of the users' channel gains.

The PDF of the i-th ordered random variable in a population of M is given by :

$$f_{X(i)}(x) = \psi_i f(x) (1 - F(x))^{M-i} F(x)^{i-1}, \qquad (6)$$

where $\psi_i = \frac{1}{B(i, M-i+1)}$, and $B(a, b)$ is the beta function defined as $B(a, b) = \frac{\Gamma(a)\Gamma(b)}{\Gamma(a+b)}$, with $\Gamma(a) = (a-1)!$. In a Rayleigh wireless environment, the channel gains, denoted by $x_i = |h_i|^2$, are exponentially distributed with PDF and cumulative density function (CDF) respectively given by $f(x) = e^{-x}$, and $F(x) = 1 - e^{-x}$. On the other hand, the joint distribution for any two order statistics, such that $x_l < x_k$, is as follows

$$f_{X(l),X(k)}(x_l, x_k) = \frac{M!}{(l-1)!(k-l-1)!(M-k)!} \times (1 - F(x))^{l-1} f(x) (F(x) - F(y))^{k-l-1} f(y) (F(y))^{M-k}. \qquad (7)$$

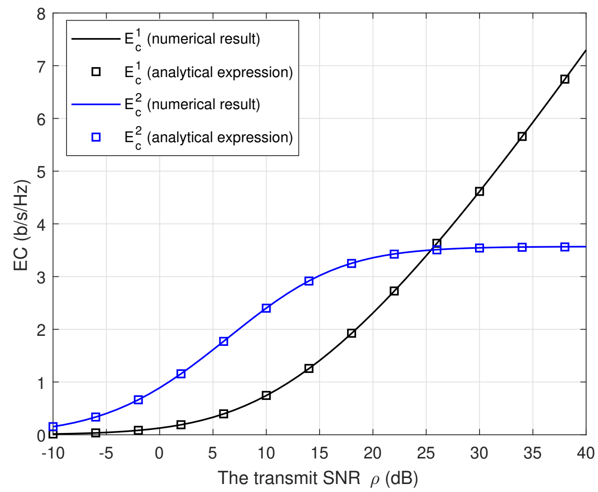

FIGURE 1 – Validation of (8) and (9) through Monte Carlo simulations. $\alpha_1 = 0.2$, $\alpha_2 = 0.8$ and $\beta_1 = \beta_2 = -1$.

As a result, the EC of U_1, denoted by E_c^1 is given by

$$\begin{aligned} E_c^1 &= \frac{1}{\beta_1} \log_2 (\mathbb{E}[(1 + \rho \alpha_1 x_1)^{\beta_1}]) \\ &= \frac{1}{\beta_1} \log_2 \left(\int_0^\infty (1 + \rho \alpha_1 x_1)^{\beta_1} f_{X(1)}(x_1) dx_1 \right) \\ &= \frac{1}{\beta_1} \log_2 \left(\frac{2}{\alpha_1 \rho} \times U \left(1, 2 + \beta_1, \frac{2}{\rho \alpha_1} \right) \right), \end{aligned} \qquad (8)$$

with $U(.,..)$ is the confluent hypergeometric function.

Similarly, the EC of U_2 is evaluated as

$$\begin{aligned} E_c^2 &= \frac{1}{\beta_2} \log_2 \left(\int_0^\infty \int_{x_1}^\infty \left(1 + \frac{\rho \alpha_2 x_2}{1 + \rho \alpha_1 x_1} \right)^{\beta_2} \right. \\ &\left. \times f_{X(1),X(2)}(x_1, x_2) dx_2 dx_1 \right) \\ &= \frac{1}{\beta_2} \log_2 \left(2 \alpha_2^{1-\beta_2} (\rho \alpha_2)^{\beta_2} e^{\frac{1}{\rho \alpha_2}} e^{-\frac{(\alpha_1 - \alpha_2)}{\rho \alpha_2}} \right) \\ &+ \frac{1}{\beta_2} \log_2 \left(\sum_{j=0}^{-\beta_2} \binom{-\beta_2}{j} (\rho \alpha_1)^j \times \sum_{k=0}^{\infty} \frac{(-1)^k (\alpha_2 - \alpha_1)^k}{k!(1+j+k)} \right. \\ &\times \left[\Gamma[2 + \beta_2 + j + k, \frac{1}{\rho \alpha_2}] \right. \\ &\left. \left. - (\rho \alpha_2)^{-1-j-k} \Gamma[1 + \beta_2, \frac{1}{\rho \alpha_2}] \right] \right), \end{aligned} \qquad (9)$$

with $\Gamma(\cdot, \cdot)$ denoting the incomplete Gamma function [6].

Proposition 1 *In a two-user NOMA uplink network, in Rayleigh fading, the closed-form expressions for EC of U_1 and U_2 are respectively given by* (8) *and* (9) *for integer β.*

The proof is omitted due to space limitation.

III.2 Asymptotic analysis : NOMA vs OMA

We make here an asymptotic analysis to compare NOMA and OMA performance, using (4) and (5). The results of this analysis is presented in the following Lemma.

Lemma 1 *In the extreme SNR regimes, given a fixed power allocation, the following conclusions hold :*

1. *For low SNRs :* $E_c^1 \to 0$, $E_c^2 \to 0$, $\widetilde{E}_c^1 \to 0$, $\widetilde{E}_c^2 \to 0$, $E_c^1 - \widetilde{E}_c^1 \to 0$, $E_c^2 - \widetilde{E}_c^2 \to 0$*;*
2. *For high SNRs :* $E_c^1 \to +\infty$, $E_c^2 \to \frac{1}{\beta_2}\log_2\left(\mathbb{E}\left[\left(1+\frac{\alpha_2|h_2|^2}{\alpha_1|h_1|^2}\right)^{\beta_2}\right]\right)$, $\widetilde{E}_c^1 \to +\infty$, $\widetilde{E}_c^2 \to +\infty$, $E_c^1 - \widetilde{E}_c^1 \to +\infty$, $E_c^2 - \widetilde{E}_c^2 \to -\infty$.

Lemma 1 indicates that the ECs of both users are vanishingly small at low SNRs, irrespective of employing NOMA or OMA. Moreover, at high SNRs, we notice that the performance of the strong user U_2 with NOMA is limited to a finite value, i.e., NOMA underperforms compared to OMA. On the contrary, for the weak user U_1, when $\rho >> 1$, its achievable EC with NOMA increases without bound. This is the exact opposite of the downlink scenario, where it is the weak user which is limited in terms of EC, at high SNRs [6].

III.3 Optimal power allocation

We formalize the optimal power allocation problem as follows

$$[\mathbf{P1}] \max_{\alpha_1,\alpha_2} \quad E_c^1 + E_c^2 \tag{10}$$

$$\text{s.t.} \quad E_c^1 \geq \widetilde{E}_c^1, \tag{11}$$

$$E_c^2 \geq \widetilde{E}_c^2, \tag{12}$$

$$0 < \alpha_1 < 1, \tag{13}$$

$$0 < \alpha_2 < 1, \tag{14}$$

The aim is to maximizes the sum EC, while ensuring to each NOMA user at least the performance they would achieve with OMA, as indicated in (11) and (12).

[**P1**] can be transformed into a single optimization problem as we have $\alpha_1 + \alpha_2 = 1$. The objective function (10) becomes an increasing function of α_2. Thus the optimal solution can directly be obtained from the constraints, particularly from (11) which gives the lower bound of α_1 (therefore the upper bound of α_2). So, solving [**P1**] boils down to solving (11) with equality.

Unfortunately, because of the complexity of (8) we cannot use it to solve analytically (11) and get the optimal power allocation for the whole range of SNRs. Instead, we do so for the extreme SNR regions.

III.3.1 At low SNRs

Using Maclaurin's series of E_c^1, we get the low-SNR approximation of the E_c^1

$$E_c^1 \approx \bar{C_w} + \rho\dot{C_w} + \frac{\rho^2}{2}\ddot{C_w} = \alpha_1 \frac{2\rho}{\ln 2}\Gamma(2)\,\mathbb{U}(2,3,2) + \alpha_1^2 \frac{\rho^2}{\ln 2}\left(\frac{\beta_1}{2} - 1\right)\Gamma(3)\,\mathbb{U}(3,4,2)\,. \tag{15}$$

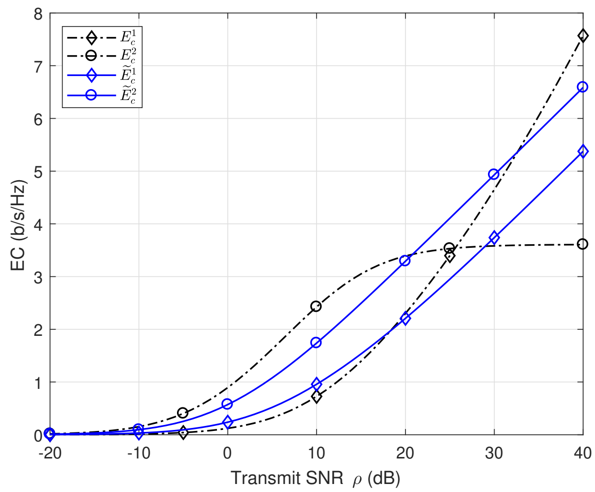

FIGURE 2 – E_c^1, E_c^2, $\widetilde{E}_c^1$ and $\widetilde{E}_c^2$ versus ρ, for fixed power allocation $\alpha_1 = 0.2$ and $\alpha_2 = 0.8$; $\beta_1 = \beta_2 = -1$.

where $\bar{C_w} = E_c^1|_{\rho=0} = 0$; $\dot{C_w} = \frac{\partial E_c^1}{\partial \rho}|_{\rho=0}$; and $\ddot{C_w} = \frac{\partial^2 E_c^1}{\partial \rho^2}|_{\rho=0}$.

Using (15), solving $E_c^1 = \widetilde{E}_c^1$ gives the optimal power allocation in low SNRs as stated in the following Proposition.

Proposition 2 *In a two-user NOMA uplink network, the optimal power allocation that maximizes the sum EC while ensuring each user to have at least the OMA performance is given as follows in low SNRs :*

$$\alpha_1^{opt} \approx -2\sqrt{\frac{1 + (2\beta_1 - 4)\ln 2\widetilde{E}_c^1}{\rho^2(\beta_1 - 2)^2}} - \frac{2}{\rho(\beta_1 - 2)}, \tag{16}$$

$$\alpha_2^{opt} \approx 1 - \alpha_1^{opt}. \tag{17}$$

III.3.2 At high SNRs

On the other hand, at high SNRs, E_c^1 can be approximated as follows

$$E_c^1 \approx \frac{1}{\beta_1} \times \log_2\left((\rho\alpha_1)^{\beta_1} 2\Gamma(1+\beta_1)\, U(1+\beta_1, 2+\beta_2, 2)\right). \tag{18}$$

(18) is used to solve $E_c^1 = \widetilde{E}_c^1$ and find the sub-optimal power allocation at high SNRs as stated in the following proposition.

Proposition 3 *In a two-user NOMA uplink network, the optimal power allocation that maximizes the sum EC while ensuring each user to have at least the OMA performance is given as follows in the high SNRs regime :*

$$\alpha_1^{opt} \approx \sqrt[\beta_1]{\frac{2^{\beta_1 \widetilde{E}_c^1}}{2\rho^{\beta_1}\Gamma(1+\beta_1)\, U(1+\beta_1, 2+\beta_2, 2)}}, \tag{19}$$

$$\alpha_2^{opt} \approx 1 - \alpha_1^{opt}. \tag{20}$$

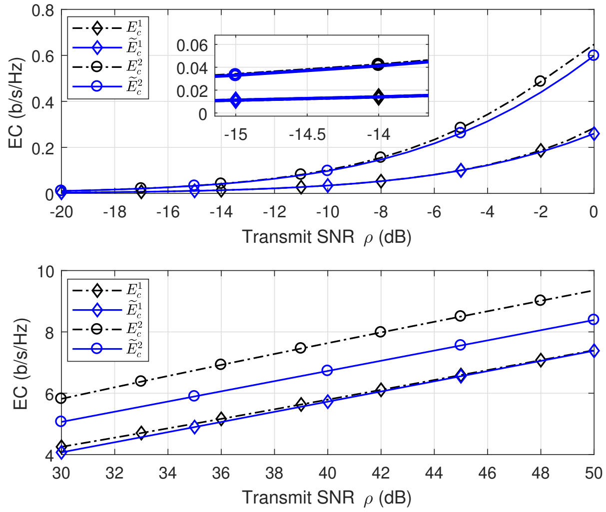

FIGURE 3 – E_c^1, E_c^2, $\widetilde{E}_c^1$ and $\widetilde{E}_c^2$ versus the transmit SNR with the optimized power allocation. (a) validation of (16) and (17). (b) validation of (19) and (20).

IV Numerical results

In this section, we validate the Lemma and the Propositions through simulations.

In Fig.1 we provide the numerical validation of the proposed closed-form expressions (8) and (9) through Monte Carlo simulations.

In Fig. 2 NOMA is compared to OMA in terms of the EC. With fixed power coefficients, $\alpha_1 = 0.2$ and $\alpha_2 = 0.8$, we note that for the weak user U_1, OMA is more advantageous than NOMA for low-medium transmit SNRs, and NOMA outperforms OMA at high transmit SNRs. Reverse conclusions can be drawn for the strong user U_2. The fact that E_c^2 converges while E_c^1 increases without bound at high SNRs is due to the interference that U_2 suffers from U_1 as its message is decoded first at the BS. This provides numerical validation for Lemma 1. What emerges is that, with a fixed power coefficients, NOMA is not always better than OMA.

Fig.3 validates the proposed optimal power allocation for the MAC layer at low and high SNRs. We remark that with the optimal power coefficients, NOMA outperforms OMA in high SNRs for U_2 and in low SNRs for U_1, where it was underperforming with fixed power coefficients. In particular the performance floor of E_c^2 in high SNRs vanished. This figure shows the importance of optimizing the power allocation at the MAC layer, and validates the Propositions 2 and 3.

Moreover, in (19) the term inside $\Gamma(.)$ should be positive, i.e., $\theta < \ln 2$; for more stringent delays beyond that threshold, not all the constraints in [**P1**] are met in high SNRs. In other words, reducing the delay beyond a certain threshold is at the expense of the individual performance gain of NOMA compared to OMA.

V Conclusion

We investigated the EC in a two-user NOMA uplink network, assuming a Rayleigh fading channel. We derived novel closed-form expressions for the ECs of the two users. We showed that, with fixed power coefficients, NOMA does not always outperforms OMA. The proposed power allocation policy allows NOMA to always win over or equal OMA for both users, but for high SNRs at the expense of delay reduction.

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