Sequential filtering of ultrasonic signals based on Gabor representation with application to rail inspection

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Résumé – Cet article présente une méthodologie pour l'étude séquentielle des signaux ultrasonores mesurés sur des rails. L'approche se base sur une décomposition parcimonieuse en ondelettes de Gabor, couplée à du filtrage statistique sur Ensembles Finis Aléatoires . Un exemple simulé montre la viabilité de la méthode, avant d'appliquer celle-ci à des signaux réels obtenus sur un rail endommagé en laboratoire, à partir de capteurs générant des ondes de surface de Rayleigh.

Abstract – In this paper, we present a new methodology to process sequentially raw measurements from ultrasonic sensors with application to rail inspection. A decomposition of signals into sets of Gabor wavelets is followed by an association step based on Random Finite Sets with the Gaussian Mixture Probability Hypothesis Density filter. The rail geometry allows estimation of the velocity of ultrasonic waves. A reprentative simulated set of samples is processed to demonstrate the benefit of this methodology. Real measurements from a laboratory machined rail are then inspected to recover the characteristics of Rayleigh ultrasonic waves.

1 Introduction

Ultrasonic Non Destructive Testing (NDT) rail analyses rely on raw measurements from a piezoelectric transducer called A-scans. Aggregation of such data forms maps called B-scans [10], from which anomalies can be detected, like flaws, reflections or attenuations [1]. Visual inspection is only possible for low rates of acquisitions. At higher rates, thousands of A-scans need to be analysed in a limited amount of time. This problem arises in modern rail inspection. Automatic analysis of individual A-scans has been studied in the NDT literature, using signal processing techniques [9], but joint analysis of A-scans has not been investigated.

Efficient methods to extract pulses features from one acquisition already exist. Sparse decomposition [5] with a Gabor dictionary is a powerful method to represent one A-scan by a set of several vectors. However, information about the evolution of this representation, for instance propagation direction or velocity, is lost. Purpose of filtering processes is to estimate these hidden states from noisy measurements. Such methodology has already been successfully applied to various types of data: for visual, spectral and cell tracking [6, 11]. Data is represented on a small dimensional space. We show how an adequate method to decompose such real signals can act as a detector. Its results are filtered by an advanced multi-target tracking algorithm, to jointly estimate the hidden states of several phenomena, here the vectors representing wave pulses, and compute their paths across A-scans.

In the present article, we introduce first the method to decompose each ultrasonic signal into a set of Gabor wavelets. The filtering method for sets is next described. From a knowledge of these pulses behaviours across acquisitions, we build a model to estimate the hidden states of interest. A synthetic dataset is created to quantify our method performances, before application on laboratory measurements performed on a rail.

2 Sparse representation of ultrasonic signals

In ultrasonic testing on non-dispersive and homogeneous materials, a Gabor dictionary allows an efficient representation of signals using a very limited number of atoms of the dictionary [2]. Additive noise or specific excitations due to the environment will generally not respect this property. Dictionary methods are therefore useful in low Signal to Noise Ratio (SNR) situations, to extract features of interest. Gabor wavelets are expressed as the product of a cosine and a Gaussian function. The values u, f, s, A and ϕ are the arrival time, central frequency, spread, amplitude and phase of the Gabor wavelet. If we define $\gamma = [u, f, s, A, \phi]^t$ as the vector describing a Gabor wavelet $g_\gamma(t)$, $t \in \mathbb{R}$, we write:

$$g_\gamma(t) = A \exp\left(-\left(t-u\right)^2/s\right) \cos\left(2\pi f t + \phi\right) \tag{1}$$

A signal y is said to be sparse on a Gabor dictionary if only few of its elements need to be used to represent this signal, if M vectors $\gamma^{(1)}$, ..., $\gamma^{(M)}$ exist such that:

$$y(t) \approx \sum_{m=1}^{M} g_{\gamma^{(m)}}(t) \tag{2}$$

The Matching Pursuit (MP) algorithm 1 aims to solve sequentially the decomposition equation 2 [5]. It searches iteratively in a dictionary D the atom with the higher inner product with y (in practice in the sub-dictionary $\tilde{D}$ with unit atoms with parameter $\tilde{\gamma} = [u, f, s]^t$), and removes it to create a residual r, used as the new signal. An optimization procedure (e.g. Gauss-Newton), is applied to fine-tune the atom. In our application, we manipulate discrete signals y of size $N \in \mathbb{N}$. The sampling frequency is f_s, and the time interval separating each point of an acquisition is thus $\Delta t = 1/f_s$.

Algorithm 1: Matching Pursuit algorithm

Input: y, M
Output: $\{\gamma^{(m)}\}_{1\leq m\leq M}$
Initialization: $m = 1$, $r = y$;
while $m \leq M$ **do**
  $\tilde{\gamma}^\star = \arg\max_{\tilde{\gamma}\in\tilde{D}} |\langle r, g_{\tilde{\gamma}}\rangle|$;
  $\gamma^\star = [\tilde{\gamma}^{\star\,t}, A^\star, \phi^\star]^t$ with $A^\star$, $\phi^\star$ from $\langle r, g_{\tilde{\gamma}^\star}\rangle$;
  $\gamma^{(m)} = \arg\max_{\gamma\in D} \|r - g_\gamma\|_2^2$ (starting from $\gamma^\star$);
  $\{\gamma^{(l)}\}_{1\leq l\leq m} \leftarrow \{\gamma^{(l)}\}_{1\leq l\leq m-1} \cup \{\gamma^{(m)}\}$;
  $r \leftarrow r - g_{\gamma^{(m)}}$;
  $m \leftarrow m + 1$;
end

Ultrasonic measurements give $K \in \mathbb{N}$ acquisitions along the rail, each consisting of a vector y_k, for $1 \leq k \leq K$. Each is decomposed into a sum of M atoms. We note Γ_k the set of Gabor parameters obtained from the decomposition of acquisition y_k, with $\Gamma_k = \{\ \gamma_k^{(m)} \mid 1 \leq m \leq M\}$.

Each Gabor vector $\gamma_k^{(m)}$ belongs to the state space $\mathcal{G} = \mathbb{R} \times \mathbb{R}_+ \times \mathbb{R}_+ \times \mathbb{R}_+^* \times [0, 2\pi[$ where $\mathbb{R}$, $\mathbb{R}_+$ and $\mathbb{R}_+^*$ denotes the set of real, positive real number and strictly positive real numbers. The sequence of measurements $(y_k)_{1\leq k\leq K}$ is therefore replaced by a sequence of sets. However, these sets can be corrupted with wrong detections made by the MP algorithm, called clutters, due to background noise.

3 Filtering with Random Finite Sets

First, one need to filter the set of atoms, to remove abnormal detections, named the clutters, to keep the wave pulses of interest, called the targets. In addition, it is necessary to perform associations between vectors of each step, to estimate the hidden variables. If a knowledge of the evolution of the Gabor vectors through acquisitions is available, for instance from physical theories, associations between vectors of Γ_{k-1} and Γ_k are possible. In the following, instead of the vector γ, we will drop the phase ϕ, to constitute the vector $z = [u, A, f, s]^t$. Sets Γ_k are thereby replaced by the sets Z_k.

A simple solution to the association problem is to minimize distances between vectors [6]. At each step an assignment matrix C is computed, with

$$C_{ij} = w_u\big(u_{k-1}^{(i)} - u_k^{(j)}\big)^2 + w_A\big(A_{k-1}^{(i)} - A_k^{(j)}\big)^2 \qquad (3)$$

the cost to assign state i of step $k-1$ to state j of step k. Weights to be chosen are only w_u an w_A. The Hungarian algorithm is applied on costs above a threshold T to minimize the total cost. This methodology, referred here as the Assignment Tracker (AT), does not take into account the pulse dynamic.

Random Finite Set (RFS) theory developed by Mahler [4] provides a solution to perform filtering on sets. At each step k, the RFS formulation allows to model the appearance of new targets by the birth intensity γ_B, and the clutter process by an intensity κ. It offers a probabilistic solution to model simple phenomena arising in multi-target tracking, such as disappearance and miss-detection of targets. The problem is formulated as the estimation of a single targets set X_k producing Z_k. Each target is defined on a space $\mathcal{X}$. In our case, this space includes the hidden information about the wave pulses.

The function $D_{k|k}(x)$, defined on $\mathcal{X}$, is called the Intensity, or Probability Hypothesis Density (PHD). It is the first moment of the multi-target posterior, which is intractable in the general case. The PHD Filter [4, 13] aims to provide an estimate of $D_{k|k}(x)$ for each time k. An estimate $\hat{X}_k$ of the state set X_k can then be obtained. It has linear complexity in the numbers of measurement vectors and targets, and can be decomposed into two steps:

1. PHD Prediction: Using information of $D_{k|k}(x)$, the transition equations, the birth and survival information, we get the predicted intensity $D_{k+1|k}(x)$;
2. PHD Update: Using the measurement set Z_k, the clutter and detection information, we compute, from $D_{k+1|k}(x)$, the updated intensity $D_{k+1|k+1}(x)$.

An analytical implementation of this filter is available, under linear Gaussian assumptions: the Gaussian Mixture PHD (GM-PHD) filter. The intensity can then be described as the sum of Gaussian components, allowing to use the Kalman equations to compute the predicted and updated intensities [13]. This filtering process is the one used in this article, applied on the set provided by the MP algorithm, which acts as a detector. An improvement of the method allows an adaptive filter [3] using an uniform birth intensity. The association of filtered states is performed in a simple way, thanks to a tag given to each individual Gaussian component, and management of the tag through iterations [7]. After each update, the tags of states are extracted to produce the track identities $\mathcal{T}$. Our complete methodology is described in algorithm 2.

Algorithm 2: Iteration of Gabor wavelets Tracker

Input: y_k, M, $D_{k-1|k-1}(x)$
Output: $\hat{X}_k$, $\mathcal{T}_k$
$Z_k \leftarrow$ MP(y_k, M);
Adapt birth intensity b_k with Z_k;
$D_{k|k-1}(x) \leftarrow$ Predict$(D_{k-1|k-1}(x), b_k)$;
$D_{k|k}(x) \leftarrow$ Update$(D_{k|k-1}(x), Z_k)$;
$\hat{X}_k \leftarrow$ StateExtraction$(D_{k|k}(x))$;
$\mathcal{T}_k \leftarrow$ TagExtraction$(D_{k|k}(x))$;

Main advantage of RFS filtering is the possibility to perform online estimation, contrary to joint decomposition and tracking methods [6], allowing large amount of acquisitions to be processed. In addition, classical Particle Filter approaches rely on a single state vector with a huge and limited dimension. The complexity of the GM-PHD filter increases only linearly.

4 Synthetic experiment

4.1 Synthetic ultrasonic data generation

We first apply the estimation process on synthetic data. We model the one dimensional propagation of surface wave pluses of velocity v on a rail. An ultrasonic emitter (E) and a receiver (R), separated by a distance d, are moved along the rail according to the x-axis between acquisitions, see figure 1. The x coordinate of emitter is noted by the variable x, which takes values indexed by the integer k such that $x_k = x_{k-1} + \Delta x$,

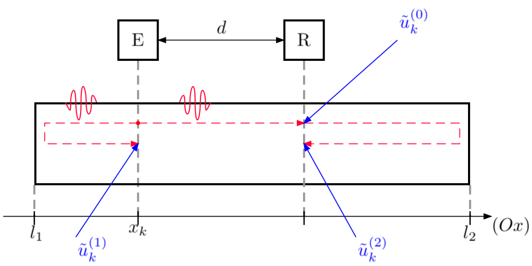


Figure 1 – Rail representation with the emitter (E) - receiver (R) system.

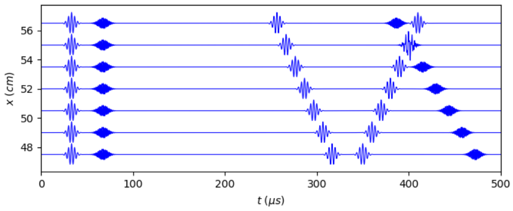


Figure 2 – Example of simulated B-scan.

with $x_0 = 10\,\mathrm{cm}$ and $\Delta x = 1.5\,\mathrm{cm}$. The emitter generates two pulses which propagate in opposite directions in the medium. A pulse is assumed to be reflected by each border of the rail, $l_1 = 0\,\mathrm{cm}$ for the left one and $l_2 = 100\,\mathrm{cm}$ for the right one. Arrival times of a wave pulse are denoted by the variable $\tilde{u}_k^{(i)}$, with i the pulse index, defined by: 0 for the front pulse which directly arrives to the sensor, 1 for the back pulse which is reflected by the left border, and 2 for the front pulse reflected by the right border. We generate 40 acquisitions, with $f_s = 4\,\mathrm{MHz}$ and 2000 samples. The sequence $\{\tilde{u}_k^{(0)}\}_{0\leq k\leq K-1}$ should therefore contain equal values. The slope of $\{\tilde{u}_k^{(1)}\}_{0\leq k\leq K-1}$ and $\{\tilde{u}_k^{(2)}\}_{0\leq k\leq K-1}$ should equal the pace, defined as $p = \pm 2v^{-1}$.

In our experiments, a wave with $v_R = 3000\,\mathrm{m\,s^{-1}}$ and central frequency $= 0.3\,\mathrm{MHz}$ is first emitted, followed after $20\,\mathrm{\mu s}$ by a wave with $v_P = 2700\,\mathrm{m\,s^{-1}}$ and central frequency $= 0.75\,\mathrm{MHz}$. Acquisitions are corrupted with white Gaussian noises, whose standard deviations are calculated to reach SNR of 10 dB and 0 dB. Figure 2 shows several generated signals. In the following, to reduce the notation burden, unities for x coordinate, time signal and u, frequency and scale are chosen to be respectively cm, µs, MHz, and $\mathrm{\mu s}^2$.

4.2 Wave pulse modelling

For one specific wave pulse, we model its behaviour by a state vector $x = [u, p_u, A, p_A, f, s]^t$, with p_u and p_A the u-paces and A-paces related to variables u and A. In addition to u, we therefore allow linear variations of A. The observation vector z from the MP algorithm is described as $z = [u, A, f, s]^t$. All perturbations are assumed to be Gaussian, giving the following state-observation model for one target:

$$x_k = F x_{k-1} + \eta_k \tag{4}$$

$$z_k = H x_k + \epsilon_k \tag{5}$$

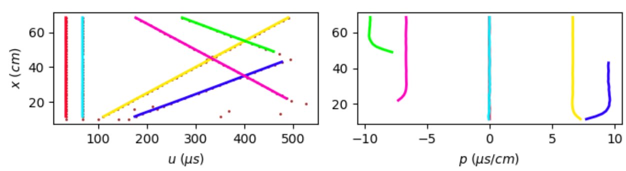
with $\eta_k \sim N(0, Q)$ and $\epsilon_k \sim N(0, R)$, where $F = \mathrm{diag}([\tilde{F}, \tilde{F}, 1, 1])$ and $Q = \mathrm{diag}([q_1\tilde{Q}, q_2\tilde{Q}, q_3, q_4])$ with intermediate matrices defined by:

$$\tilde{F} = \begin{bmatrix} 1 & \Delta x \\ 0 & 1 \end{bmatrix} \text{ and } \tilde{Q} = \begin{bmatrix} \Delta x^2/3 & \Delta x/2 \\ \Delta x/2 & \Delta x \end{bmatrix} \tag{6}$$

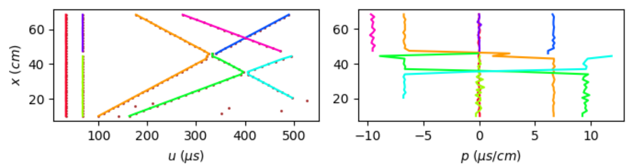
with $q_1 = q_2 = q_3 = 0.1$ and $q_4 = 10$. The observation noise is assumed Gaussian, independent for each dimension, with $R = \mathrm{diag}([\sigma_u, \sigma_A, \sigma_f, \sigma_s]) = \mathrm{diag}([10, 1, 0.1, 1])$. Probabilities of detection and survival are respectively set to 0.95 and 0.90. For each measurement, we create 3 newborn targets with hidden u-paces equal to $2/v$, $-2/v$ or 0, and A-paces to 0. The chosen value of $v = 0.27$ allows a balance between real v_R and v_P. Each newborn target has initial covariance matrix $P_B = \mathrm{diag}([1, 1, 1, 1, 1, 100])$, and weight $w_B = 1e^{-8}$. Assuming u, f, s and A restricted to intervals $[0, 500]$, $[10, 100]$, $[0.1, 1]$ and $[0, 5]$, the total space volume is therefore $V = 500 \times 0.9 \times 90 \times 5$. Lastly, the clutter rate per acquisition λ is set to 2 if $M = 5$, and 5 if $M = 10$.

An interesting fact is the possibility to calculate the multi-target likelihood of the data sets given a parameter vector θ, noted $p(Z_1, ..., Z_M \mid \theta)$. A manual calibration process is done as in this article, to find the parameter vector $\theta^\star$ which maximizes this likelihood: $\theta^\star = \arg\max p(Z_1, ..., Z_M \mid \theta)$.

The Assignment Tracker parameters are set to: $w_u = 0.01$, $w_A = 1$ and $T = 25$ for baseline results. The u-pace is computed from the tracks by differentiation.



(a) Tracks of GM-PHD Tracker



(b) Tracks of Assignment Tracker

Figure 3 – Visualization of Gabor waves trajectories from simulated B-scan.

4.3 Results

The average optimal sub-pattern assignment (OSPA) metric [12] can be used to compute the distance between real state sets and the ones estimated by the GM-PHD and Assignment Trackers. Values are averaged over all the steps. Results are gathered in table 1, for the two SNR levels. Simulations are repeated 25 times for Monte Carlo estimates.

In the high SNR scenario with limited iterations of the MP algorithm, both methods show similar results. Since the number of MP iterations M is near the real number of wavelets, no advanced filtering process is needed. Differences between the two methods appear when the level of noise increase, or when M exceeds the real number of pulses. Since the assignment method does not take into account vector dynamic, and only relies on distance calculation between vectors, it can be prone

Table 1 – Mean OSPA, averaged over 25 Monte Carlo simulations. Mean is displayed, standard deviation given in brackets, bold for higher value on each row.

MP iterations	SNR	GM-PHD	AT
5	10	**0.70** (0.0025)	0.71 (0.0064)
10	10	**0.72** (0.0036)	0.84 (0.016)
5	0	**0.76** (0.0098)	0.79 (0.012)
10	0	**0.77** (0.015)	0.88 (0.012)

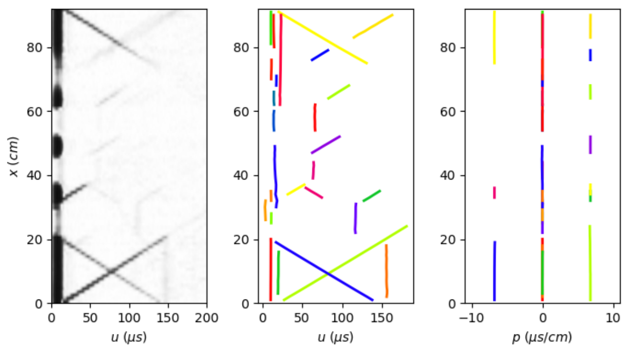


Figure 4 – Ultrasonic B-scan from machined rail (black for high absolute values) with estimates from the filtering process.

to wrong associations, and most particularly when targets are crossing. The GM-PHD tracker performs separation between clutter and real wavelets. A drawback is the delayed estimation, because the filter needs iterations to confirm a new track. Visual analysis of tracks for high SNR and $M = 5$ in figure 3 highlights this phenomenon. Brown dots represent the MP estimations. Coloured lines are the results from trackers.

5 Real data experiment

Using Electro-Magnetic Acoustic Transducers [8], a technology to generate Rayleigh surface waves on ferromagnetic materials, we inspect a $1\,\mathrm{m}$ rail. Cracks of $5\,\mathrm{mm}$ have been machines at positions 31, 46, 61 and $76\,\mathrm{cm}$. Parameters are: $d = 8\,\mathrm{cm}$, $f_s = 25\,\mathrm{MHz}$, $N = 5000$ samples, $x_0 = 0\,\mathrm{cm}$ and $\Delta x = 1\,\mathrm{cm}$. $M = 92$ signals are obtained.

Same model is applied to the data, but only visual inspection can be performed. Small reflections from cracks can easily be identified in figure 4. Left part shows the B-scan, with each row representing one A-scan, with amplitude proportional to the level of grey colour. Middle and right parts show estimations of u and u-paces. Velocity of reflected waves is near $2950\,\mathrm{m\,s^{-1}}$, which is a consistent value for a surface wave velocity.

6 Conclusion

We have presented a new methodology to analyse sequences of ultrasonic measurements thanks to a sparse decomposition scheme. Extracted components are then associated with a probabilistic filter, to estimate online their behaviour evolution. Application on ultrasonic rail testing showed the ability of this methodology to identify pulses paths and reflections on cracks, with identification of their velocity.

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